北京理工大學
BEIJING INSTITUTE OF TECHNOLOGY

复杂系统智能控制与决策
国家重点实验室培育基地

Distributed Swarming Control with Connectivity Maintenance and Obstacle Avoidance in Multi-agent Systems

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05-10-2012

Outline

1. **Preliminaries and Problem Statement**
   - Motivation
   - Graph Theory
   - Potential Research Directions
2. **Research on Distributed Swarming Control in Multi-agent System**
   - Two-Tiered Hierarchical Network Architecture
   - Distributed Motion Coordination with Connectivity Maintenance
   - Distributed Swarming Control with Obstacle Avoidance
3. **Future Work**
   - Future Work

Multi-agent Systems in Nature

What is Multi-agent System?

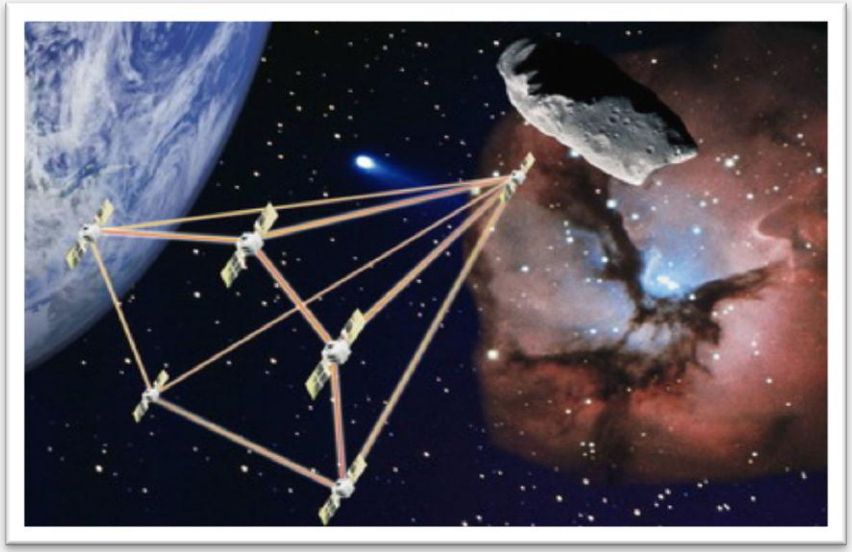
Definition

A multi-agent system (MAS) is a system composed of multiple interacting intelligent agents. Multi-agent systems can be used to solve problems that are difficult or impossible for an individual agent or a monolithic system to solve.

Important Characteristics of agents in MAS

- **Autonomy**: the agents are at least partially autonomous;
- **Local views**: no agent has a full global view of the system, or the system is too complex for an agent to make practical use of such knowledge;
- **Decentralization**: there is no designated controlling agent (or the system is effectively reduced to a monolithic system).

Multi-agent Systems in Industrial Community



How to Study Multi-agent Systems?

Basic Tool

Graph Theory

Basic Concepts of Graph Theory

Dynamic Communication Graph

The dynamic communication graph $G(t)=\{V,E(t)\}$ is a time-varying directed/undirected graph consisting of a set of N vertices $V=\{n_1,n_2,\ldots,n_N\}$ indexed by the agents of the group and an edge set $E(t)=\{(n_i,n_j)\mid \|x_{ij}(t)\|<R\}$, containing ordered/unordered pairs of nodes that represent neighboring relations.

Graph Connectivity

A dynamic graph $G(t)$ is connected at time t , if there exists a path between any two vertices in $G(t)$. Where a path means a sequence of distinct vertices such that consecutive vertices are adjacent.

Potential Research Directions of Multi-agent Systems

Objective

- Leaderless Consensus
- Swarming/Flocking with Collision Avoidance
- Formation Control (Rotation...)
- Coordinated Tracking with One Leader
- Containment Control with Multiple Leaders

Model

- First-order Integrators
- Second-order Integrators
- General Linear Systems
- Nonlinear Systems
  - Euler-Lagrange Systems !
  - Attitude Dynamic of Rigid Bodies
  - Nonholonomic Unicycles
  - General Nonlinear Systems

Issue

time delay, switching/random network, quantized, sampled-data, finite-time, output feedback, optimization, gossip, game theory, event-triggering based,…

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3 Future Work

- Future Work

Problem Formulation

A group of fully actuated agents moving in an n-dimensional Euclidean space, a continuous-time model of the system is described by

$$\dot{x}_i = v_i$$

$$\dot{v}_i = u_i \qquad i = 1, 2, \ldots, N$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is the position vector of agent i ,
$v_i = (v_{i1}, v_{i2}, \ldots, v_{in})^T \in R^n$ is the velocity vector,
$u_i = (u_{i1}, u_{i2}, \ldots, u_{in})^T \in R^n$ is the control input acting on agent i .

Objective of The Paper

Achieving stable swarming motion with connectivity maintenance and obstacle avoidance under arbitrary initially connected two-tiered hierarchical network .

Stable Swarming Motion :
It is called a stable flock when all the agents asymptotically approach the same velocity, collisions between interactive agents are avoided and the final tight configuration of the system can minimize all agent potentials.

Distributed Swarming Control in Multi-agent Systems

A Two-Tiered Hierarchical Network Architecture

B Distributed Motion Coordination with Connectivity Maintenance

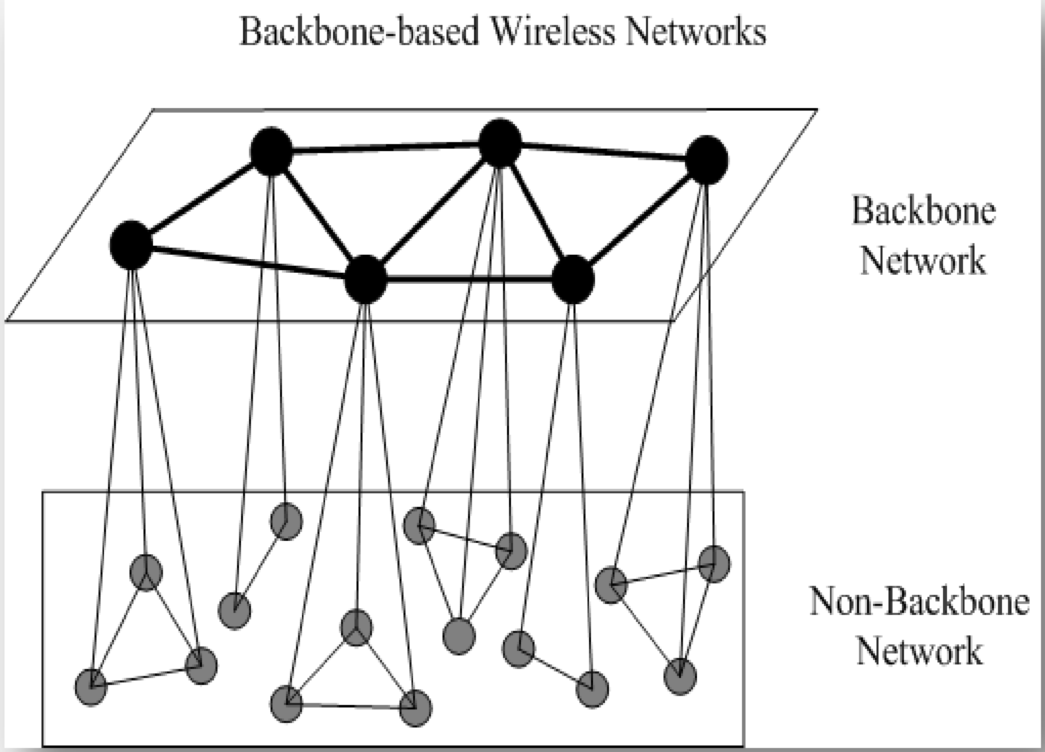
C Distributed Swarming Control with Obstacle Avoidance

Two-Tiered Hierarchical Network Architecture

Construction of the Backbone-Based Network

Distributed clustering algorithm using the weight-based heuristics:

① Node Degree ② Communication Quality ③ Node Mobility



Advantage (∼ flat network)

- Network Performance;
- Communication Overhead;
- Extensibility;
- Flexibility;
- Resource Allocation;
- Mobility Management

Two-Tiered Hierarchical Network Architecture

Weights Setting

① Node Degree

The node degree is the number of neighbors of each node.

② Communication Quality

The quality of communication link between the node i and the node j:

$$CQ_{i\leftrightarrow j} = s \times \min(RSS_{i\to j}, RSS_{j\to i})$$

$RSS_{i\to j}$ represents the received signal strength of the node i detected by the node j, s is the receiving sensitivity of the communication signal.

For any node i, its communication quality:

$$CQ_i = \frac{1}{|N_i|}\sum_{j\in N_i} CQ_{i\leftrightarrow j}$$

Two-Tiered Hierarchical Network Architecture

Weights Setting

③ Node Mobility

Node mobility is estimated by the measure of the relative motion.

The node estimates the relative movement measure between the node i and the node j using $RSS_{j\to i}$:

$$NM_{i\leftrightarrow j} = \ln \frac{RSS_{j\to i}^{t}}{RSS_{j\to i}^{t-1}}$$

The mobility of any node i is the average movement measure of all his neighboring nodes:

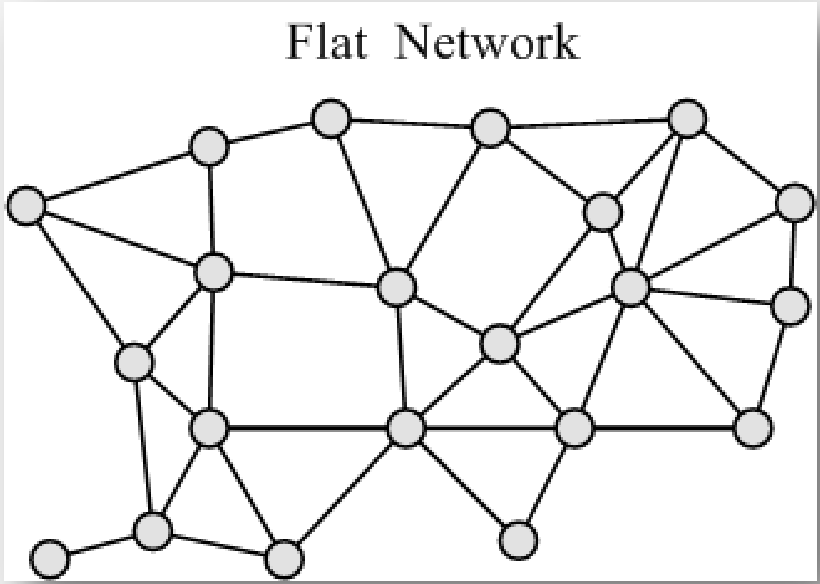
$$NM_{i} = \frac{1}{|N_i|}\sum_{j\in N_i}\left|NM_{i\leftrightarrow j}\right|$$

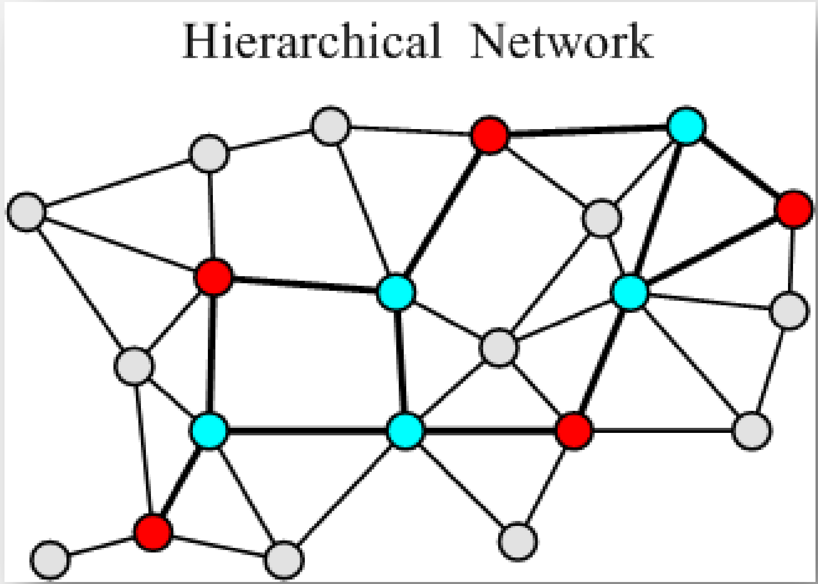
The greater the NM_i , the faster motion of the node change.

Two-Tiered Hierarchical Network Architecture

Backbone Network Connections

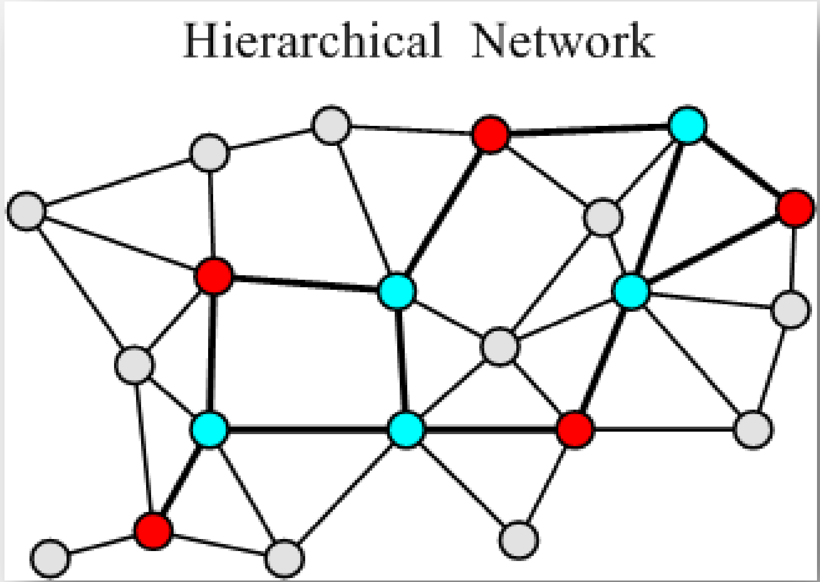
- The nodes with the maximum weight value are selected as the cluster head;
- When two neighboring clusters overlap with several nodes which can communicate directly with the two cluster heads, the node with the minimum weight value is selected as the gateway;
- when two neighboring clusters is not overlapping, the two nodes from the two neighboring clusters are selected as the distributed gateway in accordance with the principle that the sum of the weight of the two nodes is the minimum.

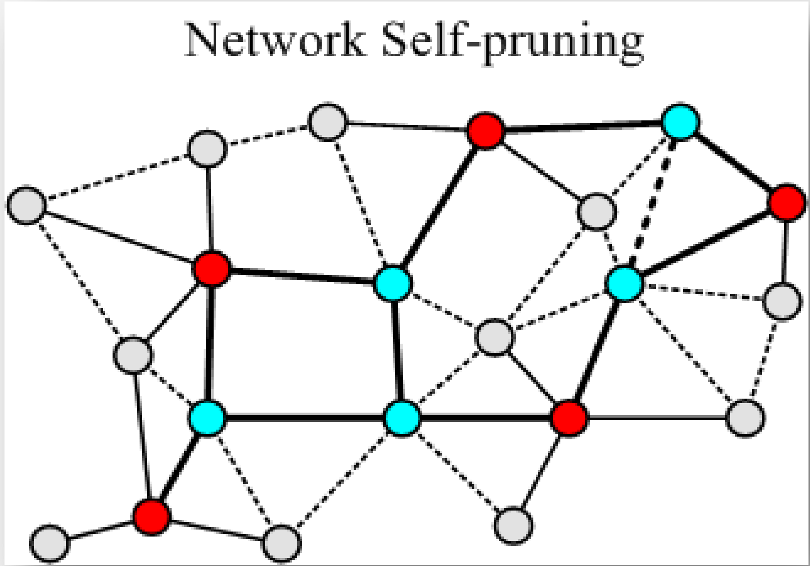




Network Connections Self- pruning

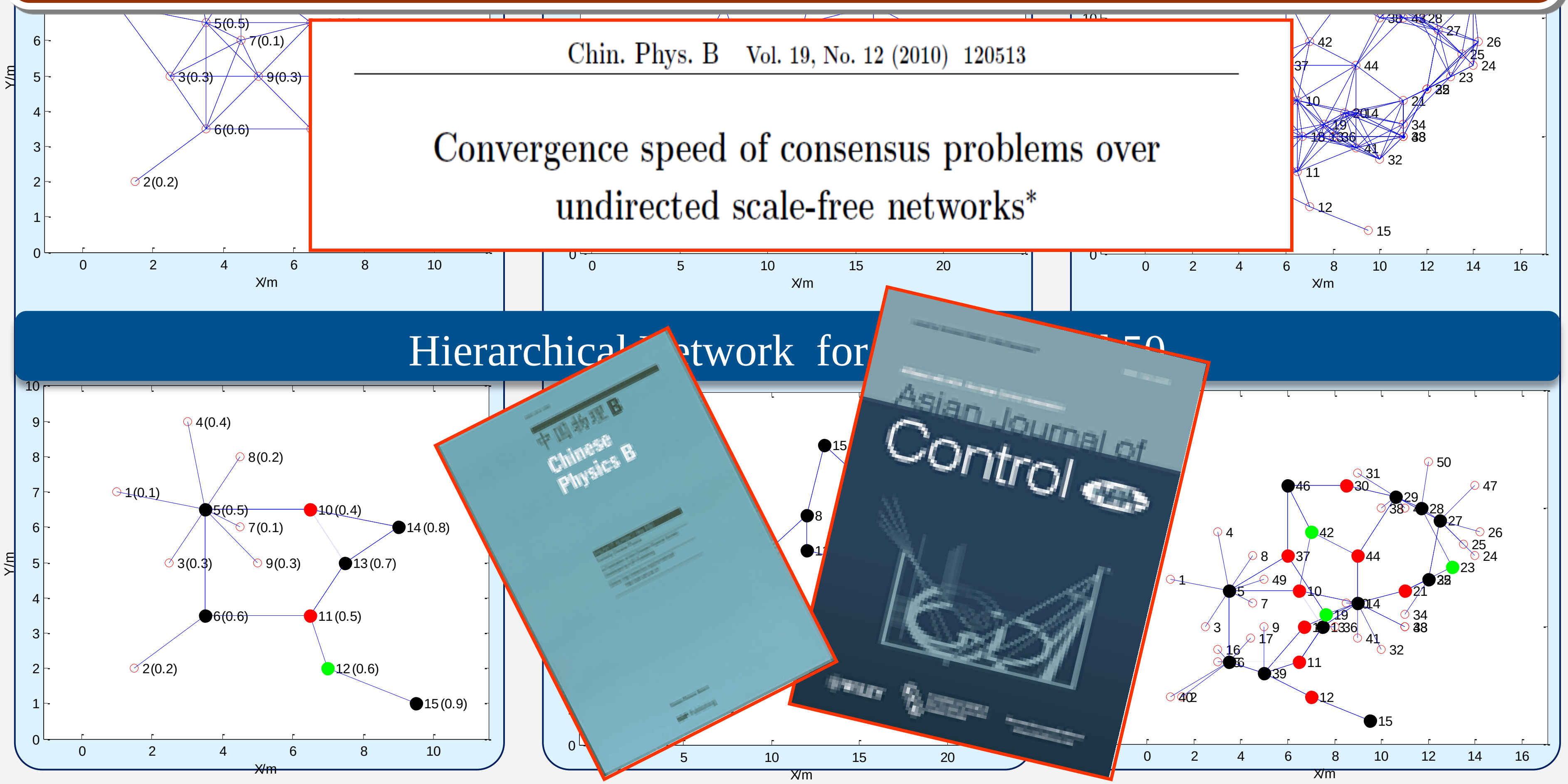
- Backbone Network: To avoid redundant connections where three backbone nodes make a triangle, the connection between two backbone nodes with lower weight will be removed from backbone connections.
- Non-Backbone Network: The non-backbone node only connects with its cluster heads, others connections will be removed.





Simulation Result

Published 2 SCI Papers in "Chinese Physics B" and "Asian Journal of Control"



A Two-Tiered Hierarchical Network Architecture

B Distributed Motion Coordination with Connectivity Maintenance

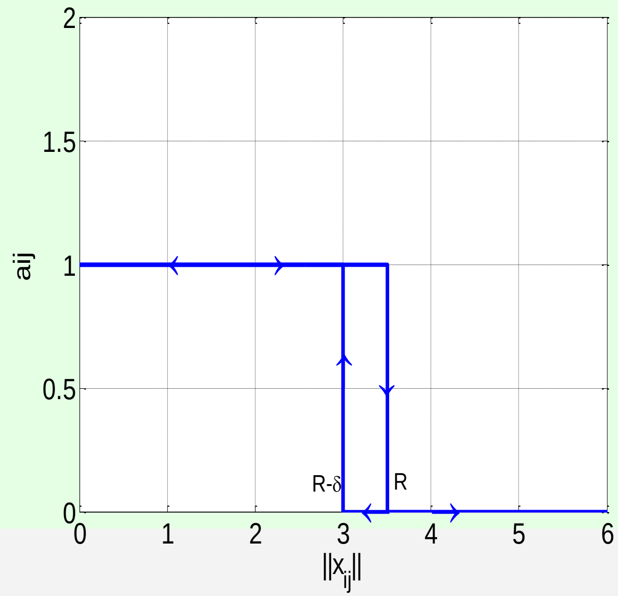
C Distributed Swarming Control with Obstacle Avoidance

Dynamic Adjacency Matrix

Time-varying neighboring relations between the agents:

- Initial links $E(0)=\{(n_i,n_j) \mid \|x_{ij}(0)\|<R, n_i,n_j \in V\}$;
- If agents *i* and *j* are not neighbors before time *t* and $\|x_{ij}(t)\|<R-\delta \ (0<\delta<R)$, then a new link is added between them;
- If $\|x_{ij}(t)\|\geq R$, then (n_i,n_j) is removed from $E(t)$.

$$A(t)=[a_{ij}(t)]=\begin{cases} 0 & \text{if } ((a_{ij}(t)=0)\wedge(\|x_{ij}(t)\|\geq R-\delta)) \\ & \quad \text{or } ((a_{ij}(t)=1)\wedge(\|x_{ij}(t)\|\geq R)) \\ 1 & \text{otherwise} \end{cases}$$



$0<\delta<R$ is a constant switching threshold.

Artificial Potential Function

Since it is required that all the agents attain a common velocity while maintaining the desired group shape, then it is desired:

$$\begin{cases} v_{ij}(t) = v_i(t) - v_j(t) \to 0 \\ x_{ij}(t) = x_i(t) - x_j(t) \to d_{th} \end{cases}$$

$$V_{ij}(\| x_{ij} \|) = \begin{cases} \left(\dfrac{1}{\| x_{ij} \|} - \dfrac{1}{d_{th}}\right)^{c_1} \dfrac{1}{(R^2 - \| x_{ij} \|^2)^{c_2}} & 0 \le \left\| x_{ij} \right\| \le R \\ c & \left\| x_{ij} \right\| > R \end{cases}$$

$V(\| x_{ij} \|)$ is a non-negative, piecewise continuous, differentiable for (0,R).

① $V(\| x_{ij} \|) \to \infty$, as $\| x_{ij} \| \to 0$ or $\| x_{ij} \| \to R$;

② V_{ij} attains its global minimum when obtains a predefined distance d_{th} .

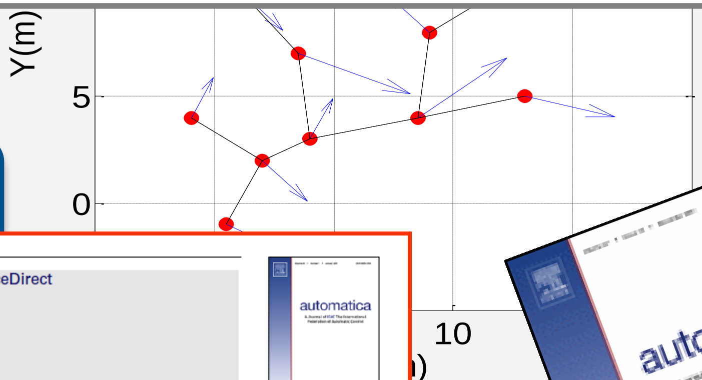
Control Law (1)

$$u_i = -\sum_{j \in N_i(t)} \nabla_{x_{ij}} V_{ij}(\| x_{ij} \|) - \sum_{j \in N_i(t)} a_{ij}(t)(v_i - v_j)$$

$N_i(t)$ is the time dependent neighborhood of agent i at time t.

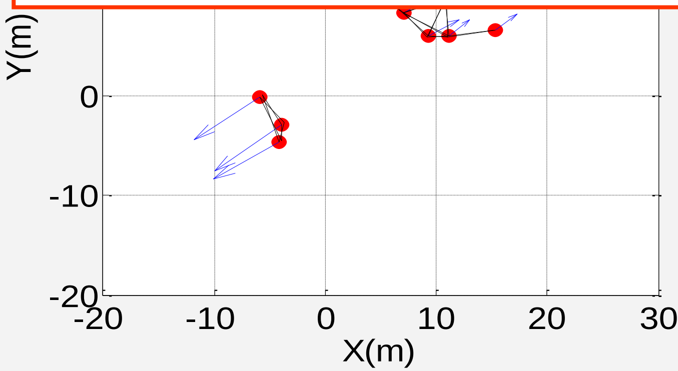
Simulation Result

① Initial State of the System



Published 3 SCI Papers in "Automatica" and "Systems & Control Letters"

② Final State of the System



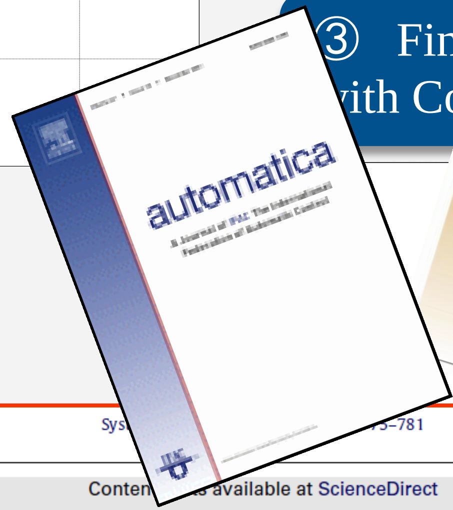
③ Final

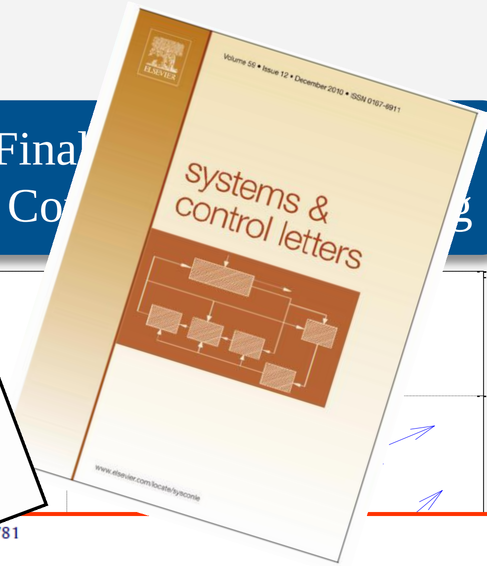
Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Connectivity preservation for multi-agent rendezvous with link failure☆





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Partial state consensus for networks of second-order dynamic agents☆

Distributed Swarming Control in Multi-agent Systems

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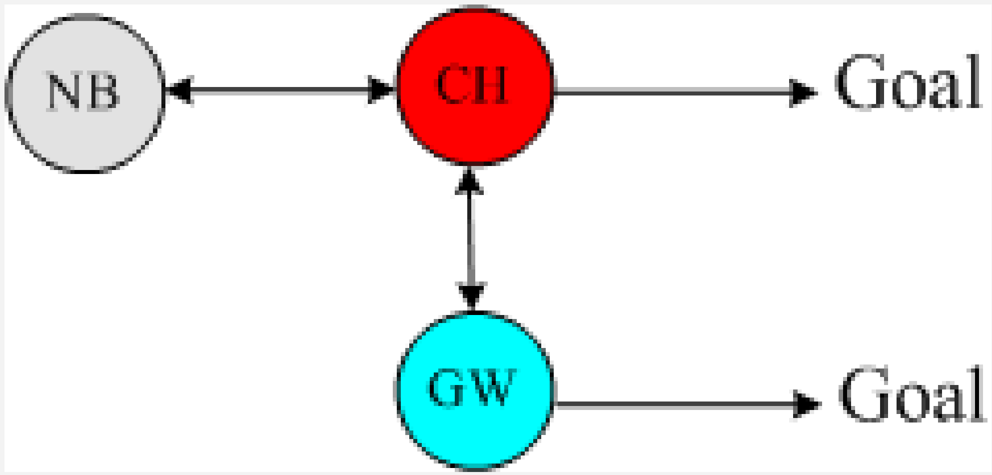
Distributed Swarming Control with Obstacle Avoidance

Motion Constraint based on Hierarchical Network

- The global network is divided into backbone network $G_B(t)=\{V_B, E_B(t)\}$ and non-backbone network $G_{NB}(t)=\{V_{NB}, E_{NB}(t)\}$;
- The backbone nodes achieve stable swarming motion while preserving the connectivity of the backbone network;
- The non-backbone nodes move following the backbone nodes.



Potential Function for Backbone Network

Connectivity maintenance:

$$V_{ij} = \frac{\gamma(x_{ij})}{(\gamma(x_{ij})^s + G_c(x_{ij}))^{1/s}}, \quad s > 0 \qquad \forall \boldsymbol{i}, \boldsymbol{j} \in \boldsymbol{G_B}$$

$$G_c(x_{ij}) = \|R\|^2 - \|x_{ij}\|^2, \qquad \gamma(x_{ij}) = \|x_{ij}\|^2 - d_{th}{}^2,$$

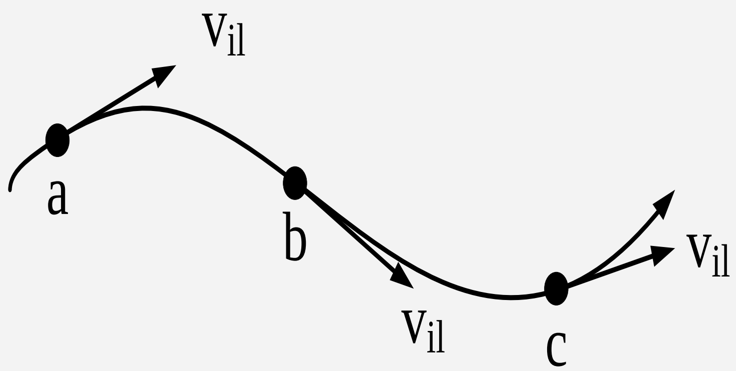
Target position:

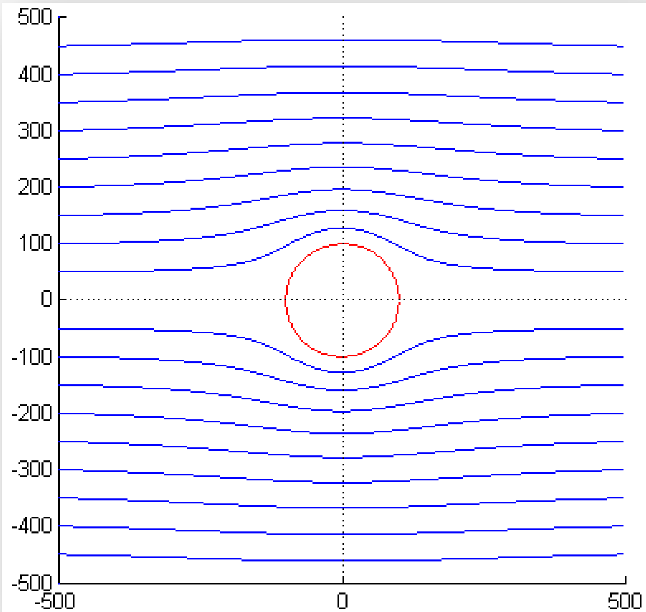
$$V_i^D = K_1 \left\| x_i - x_{des} \right\| = K_1 \left\| x_{id} \right\| \quad , \ i \in G_B$$

K_1 is the coefficient of attractive force, x_{des} is the position vector of target point.

Avoiding Obstacle for Backbone Network

- The concept of fluid mechanics would be used and virtual leaders should be introduced for each backbone node that is free of collision from obstacles.
- The basic idea is to regard the nodes as part of the flow, and take the stream lines as the reference trajectories to be followed by the nodes.





Stream Function

The components of the fluid velocity in two-dimensional plane is defined by

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}.$$

In which $\psi(x, y)$ is the stream function. The velocity of the fluid field must satisfy its continuous condition, that is,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Streamline

Streamlines are curves that show the mean direction of the fluid at the same instant of time. The curves are tangent to the velocity vectors at any points occupying on the streamline. It depicts the motion of the different particles in the flow field at the same instant of time.

Avoiding Obstacle for Backbone Network

When each backbone node enters in the fluid potential range of the obstacle, nodes select their orthogonal projection point on the streamlines as a set of the virtual leaders, the velocity of the virtual leaders are assigned with the speed v_{il} , which is equal to the velocity of the stream field around the obstacle.

$$v_l = \nabla \psi = u + iv.$$

The potential field that backbone node i moves following its virtual leader v_{il} is denoted by V_{il} which has the same form as that of V_{ij} .

Control Law for Backbone Network (2)

$$u_i = -k_1 \sum_{j \in N_B^i(t)} \nabla_{x_{ij}} V_{ij}(\| x_{ij} \|) - \sum_{j \in N_B^i(t)} a_{ij}(t)(v_i - v_j)$$

$$-\sigma_i(t)\left[k_2 \nabla_{\hat{x}_i} V_{il}(\| \hat{x}_i \|) + \hat{v}_i - \dot{v}_{il}\right] - k_3 \nabla_{x_i} V_i^D(\| x_{id} \|)$$

$$\sigma_i(t) = \begin{cases} 0, & x_{io} > R \\ 1, & 0 < x_{io} \le R \end{cases}$$

$k_1, k_2, and\ k_3$ are predefined scalar weights;

x_{io} is the distance between the backbone node and the obstacle;

$N_B^i(t)$ is the time dependent backbone neighborhood of backbone node i at time t.

$$\hat{x}_i = x_i - x_{il} \qquad \hat{v}_i = v_i - v_{il} \qquad \hat{v}_j = v_j - v_{jl}$$

Distributed Swarming Control with Obstacle Avoidance

Potential Function for Non-backbone Network

Maintaining connectivity with its cluster head and avoiding collision with all the other neighboring nodes:

$$V_{ij}^{'} = \frac{K^2}{\left\| x_{ij} \right\|^2} + \log \left\| x_{ij} \right\|^2, \quad i \in G_{NB}(t)$$

Obstacle Avoidance:

$$V_i^O = \begin{cases} \dfrac{K_2}{x_{io}}, & 0 < x_{io} \le R \\ 0, & x_{io} > R \end{cases}, \quad i \in G_{NB}(t)$$

where K_2 is the coefficient of repulsive force, x_{io} is the distance between the non-backbone node and the obstacle.

Distributed Swarming Control with Obstacle Avoidance

Control Law for Non-backbone Network (3)

$$u_i = \left[-k_4 \sum_{j \in N_i(t)} \nabla_{x_{ij}} V_{ij}^{'}(\| x_{ij} \|) - (v_i - v_j) + \dot{v}_j \right] - k_5 \nabla_{x_i} V_i^{O}(\| x_{io} \|) - \sum_{j \in N_i(t)} a_{ij}(t)(v_i - v_j)$$

k_4 *and* k_5 are predefined scalar weights;

$N_i(t)$ is the time dependent neighborhood of non-backbone node i at time t.

Distributed Swarming Control with Obstacle Avoidance

Stability Analysis

Theorem 1. Consider a system of nodes with dynamics (1), suppose the initial network $G(t_0)$ is connected. N_1 backbone nodes and N_2 non-backbone nodes are steered by control law (2) and (3), respectively. Then the desired stable swarming motion can be achieved when all the nodes asymptotically approach the same velocity.

Stability Proof

Total Lyapunov function : $J = J_1 + J_2$

Lyapunov function for Backbone Node:

$$J_1 = \frac{1}{2}\sum_{i=1}^{N_1}\left[\left(k_1 \sum_{j\in N_B^i(t)} V_{ij}(\| x_{ij} \|)\right)\right] + k_2\sum_{i=1}^{N_1}\left(V_{il}(\| \hat{x}_i \|)\right) + k_3\sum_{i=1}^{N_1} V_i^D + \frac{1}{2}\sum_{i=1}^{N_1}\left(\hat{v}_i^T \hat{v}_i\right)$$

Lyapunov function for Non-backbone Node:

$$J_2 = \frac{1}{2}\sum_{i=1}^{N_2}\left(k_4 \sum_{j\in N_i(t)} V_{ij}^{'}(\| x_{ij} \|)\right) + k_5\sum_{i=1}^{N_2} V_i^O(\| x_{io} \|) + \frac{1}{2}\sum_{i=1}^{N_2}(v_i - v_j)^T (v_i - v_j)$$

Distributed Swarming Control with Obstacle Avoidance

Stability Proof

$$J_1 = \frac{1}{2}\sum_{i=1}^{N_1}\left(k_1 \sum_{j\in N_B^i(t)} V_{ij}(\| \hat{x}_{ij} + (x_{il} - x_{jl}) \|)\right) + k_3 \sum_{i=1}^{N_1} V_i^D + k_2 \sum_{i=1}^{N_1}\left(V_{il}(\| \hat{x}_i \|)\right) + \frac{1}{2}\sum_{i=1}^{N_1}\left(\hat{v}_i^T \hat{v}_i\right)$$

$$\dot{J}_1 = \sum_{i=1}^{N_1} \hat{v}_i^T \left(k_1 \sum_{j\in N_B^i(t)} \nabla_{\hat{x}_i} V_{ij}(\| \hat{x}_{ij} + (x_{il} - x_{jl}) \|)\right) + k_3 \sum_{i=1}^{N_1} \hat{v}_i^T \nabla_{\hat{x}_i} V_i^D + k_2 \sum_{i=1}^{N_1} \hat{v}_i^T \nabla_{\hat{x}_i} V_{il}(\| \hat{x}_i \|) + \sum_{i=1}^{N_1}\left(\hat{v}_i^T (\dot{v}_i - \dot{v}_{il})\right)$$

$$\begin{aligned}
\dot{J}_1 &= \sum_{i=1}^{N_1}\left(\hat{v}_i^T\left(-\hat{v}_i - \sum_{j\in N_B^i(t)} a_{ij}(t)\left(v_i - v_j\right)\right)\right) \\
&= -\sum_{i=1}^{N_1} \hat{v}_i^T \hat{v}_i - \sum_{i=1}^{N_1}\left(\hat{v}_i^T \sum_{j\in N_B^i(t)} a_{ij}(t)\left(v_i - v_j\right)\right) \\
&= -\sum_{i=1}^{N_1} \hat{v}_i^T \hat{v}_i - \hat{v}_i^T (L_{G_B}(t) \otimes I_{N_1})\hat{v} \\
&= -\hat{v}_i^T [(H_{N_1} + L_{G_B}(t)) \otimes I_{N_1}]\hat{v}
\end{aligned}$$

Distributed Swarming Control with Obstacle Avoidance

Stability Proof

$$J_2 = \frac{1}{2}\sum_{i=1}^{N_2}\left(k_4 \sum_{j\in N_i(t)} V_{ij}^{'}(\| x_{ij} \|)\right) + k_5 \sum_{i=1}^{N_2} V_i^O(\|x_{io}\|) + \frac{1}{2}\sum_{i=1}^{N_2}(v_i - v_j)^T(v_i - v_j)$$

$$\dot{J}_2 = \sum_{i=1}^{N_2}(v_i - v_j)^T\left(k_4 \sum_{j\in N_i(t)} \nabla_{x_{ij}} V_{ij}^{'}(\| x_{ij} \|)\right)$$

$$+ k_5 \sum_{i=1}^{N_2}(v_i - v_j)^T \nabla_{x_{ij}} V_i^O(\|x_{io}\|) + \sum_{i=1}^{N_2}(v_i - v_j)^T(\dot{v}_i - \dot{v}_j)$$

$$\dot{J}_2 = \sum_{i=1}^{N_2}(v_i - v_j)^T\left(-(v_i - v_j) - \sum_{j\in N_i(t)} a_{ij}(t)(v_i - v_j)\right)$$

$$= -\sum_{i=1}^{N_2}(v_i - v_j)^T(v_i - v_j) - \sum_{i=1}^{N_2}\left((v_i - v_j)^T \sum_{j\in N_i(t)} a_{ij}(t)(v_i - v_j)\right)$$

$$= -(v_i - v_j)^T[(H_{N_2} + L_{G_{NB}}(t)) \otimes I_{N_2}]\hat{v}$$

$$\dot{J} = \dot{J}_1 + \dot{J}_2 \le 0 \quad , \quad \frac{d\| x_{ij} \|^2}{dt} = 2x_{ij}{}^T\dot{x}_{ij} = 2x_{ij}{}^T(v_i - v_j) = 0$$

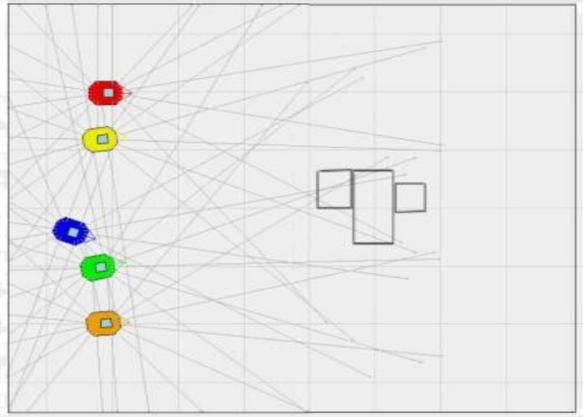
Distributed Swarming Control with Obstacle Avoidance

Simulation Result

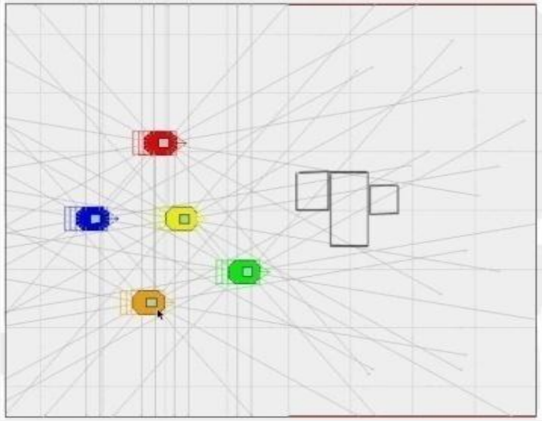
Obstacle Avoidance Algorithm Using Stream Function

Initial State

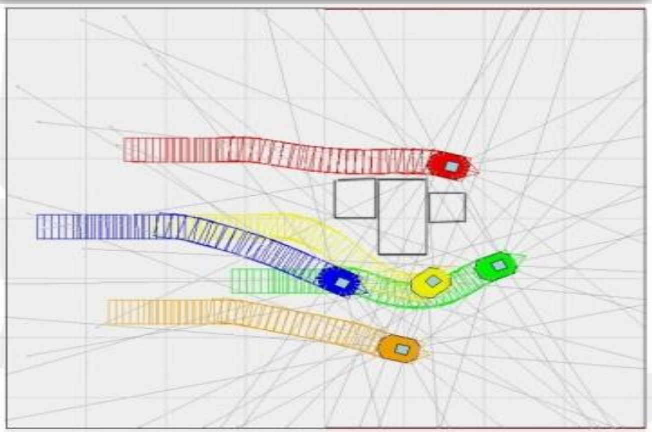
Position Disarrangement



Formation Control



Obstacle Avoidance Control



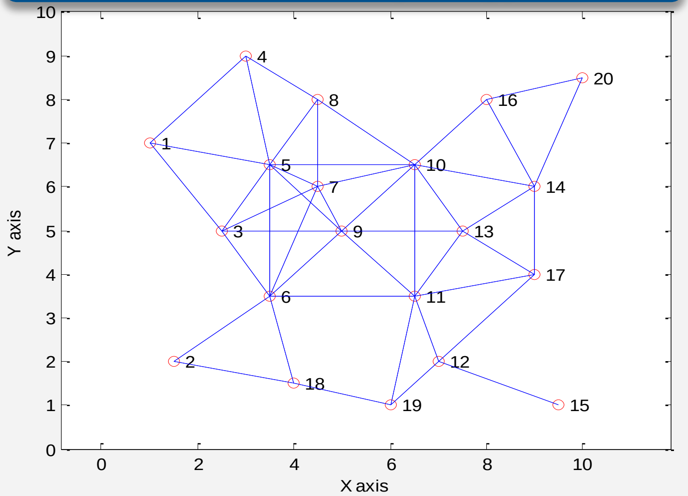
Obstacle Avoidance Completion

Distributed Swarming Control with Obstacle Avoidance

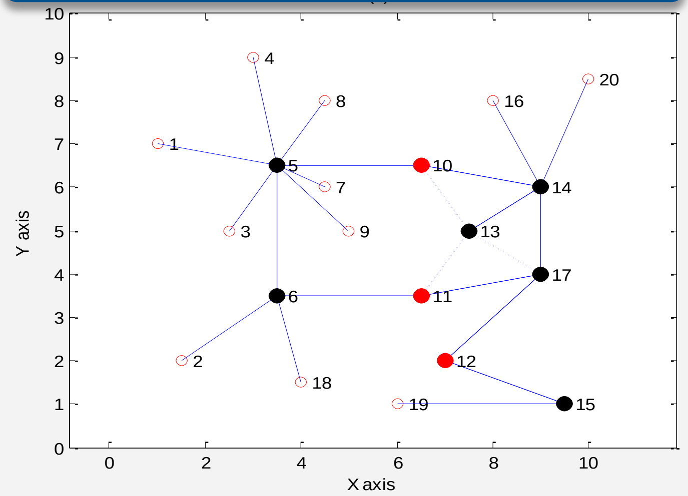
Simulation Result

Swarming Control Algorithm

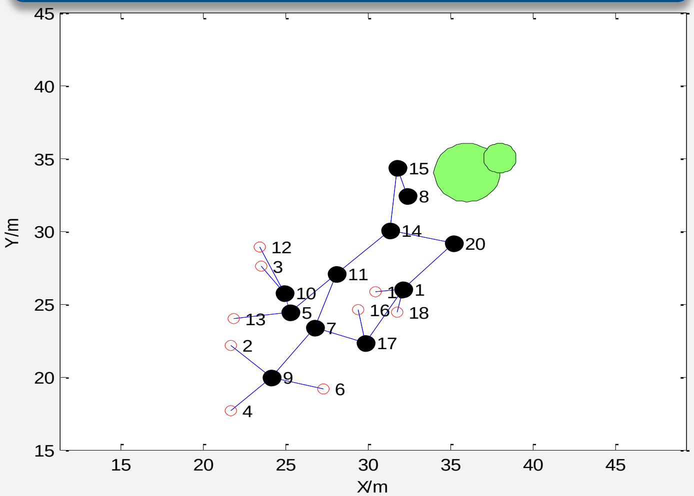
Initial Network Topology at t=0



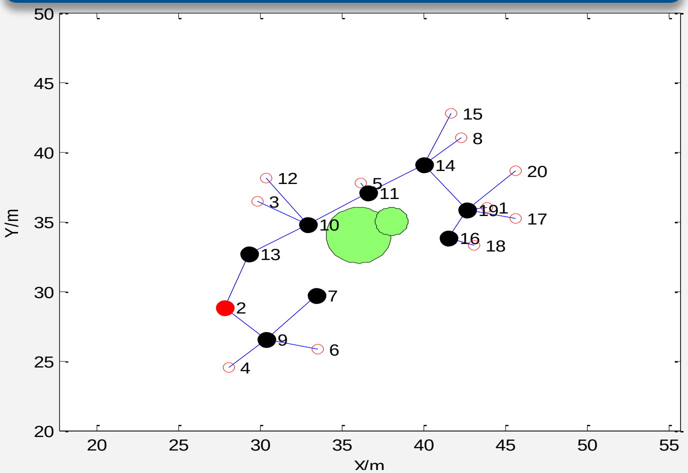
Initial hierarchical network



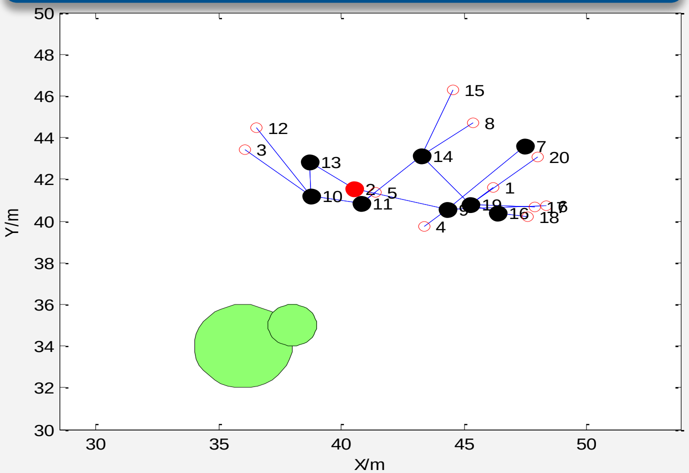
t=5s



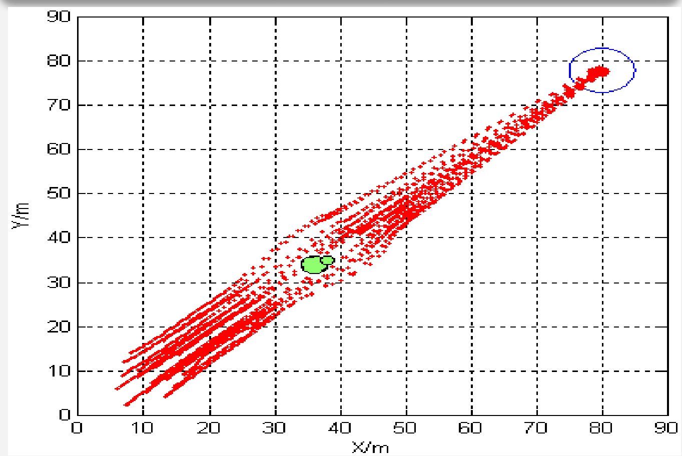
t=10s



t=15s



Entire Trajectory t=0～20s

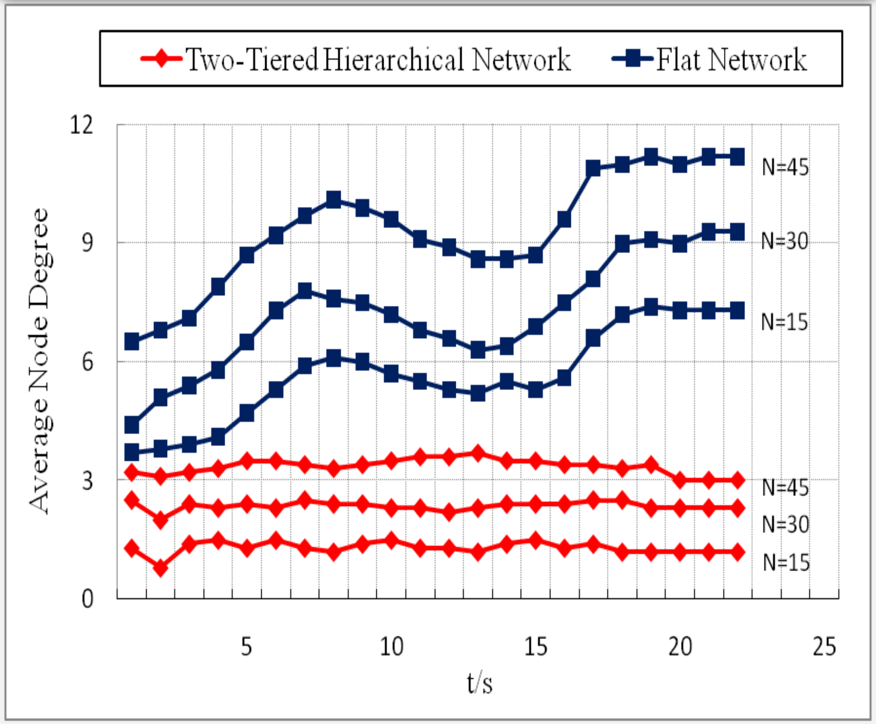


Distributed Swarming Control with Obstacle Avoidance

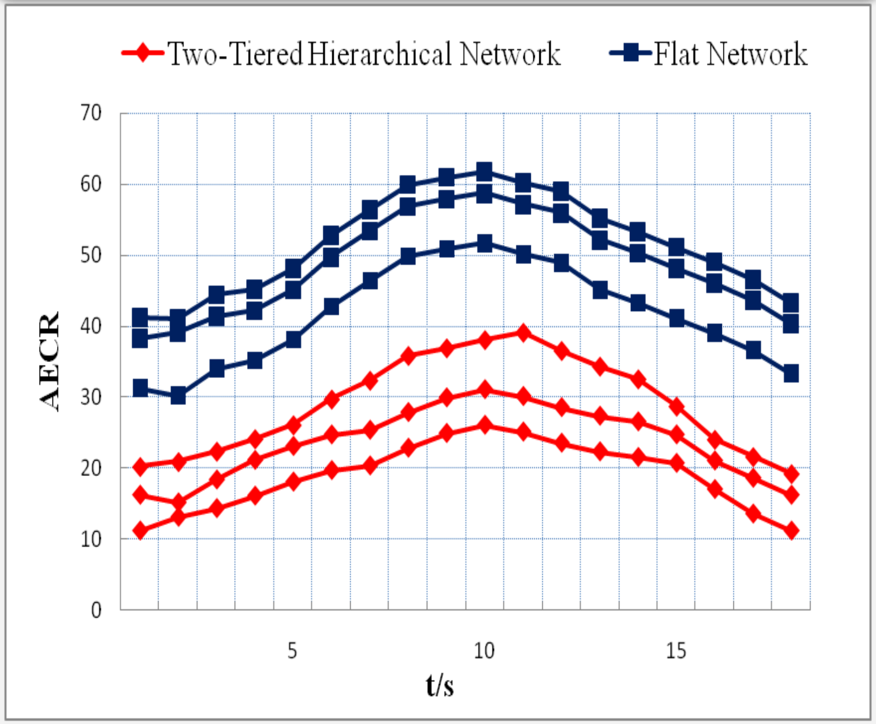
Simulation Result

Communication Cost between Two-Tiered Hierarchical Network and Flat network

Average Node Degree



Average Energy Consumption Ratio



Distributed Swarming Control with Obstacle Avoidance

Simulation Result

Real Object Simulation

One paper co-authored with Prof. Wei Ren is accepted by "Systems & Control Letters". Another paper is submitted to CDC 2012

t=35s

t=45s

Outline

1 Preliminaries and Problem Statement

- Motivation
- Graph Theory
- Potential Research Directions

2 Research on Distributed Swarming Control in Multi-agent System

- Two-Tiered Hierarchical Network Architecture
- Distributed Motion Coordination with Connectivity Maintenance
- Distributed Swarming Control with Obstacle Avoidance

3 Future Work

- Future Work

Future Work

Future Work

- Swarming/Flocking with fault-tolerance network topology；
- Coordination for multiple Lagrangian systems with both absolute and relative velocity measurements under directed graphs;
- Swarming/Flocking for multiple Lagrangian systems with obstacle and collision avoidance.

Above works are supported by Projects of Major International (Regional) Joint Research Program NSFC（61120106010）, National Science Fund for Distinguished Young Scholars，No. 60925011

Thank You!

Any Question?