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Distributed Nonlinear Output Regulation for the Coordinative Control of Multi-agent Systems

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Outline

1. Introduction
2. The introduction for output regulation theory
3. The output regulation for multi-agent systems
4. The robust output regulation for multi-agent systems
5. Conclusions

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1. Introduction

Systems consists of multiple agents

communication

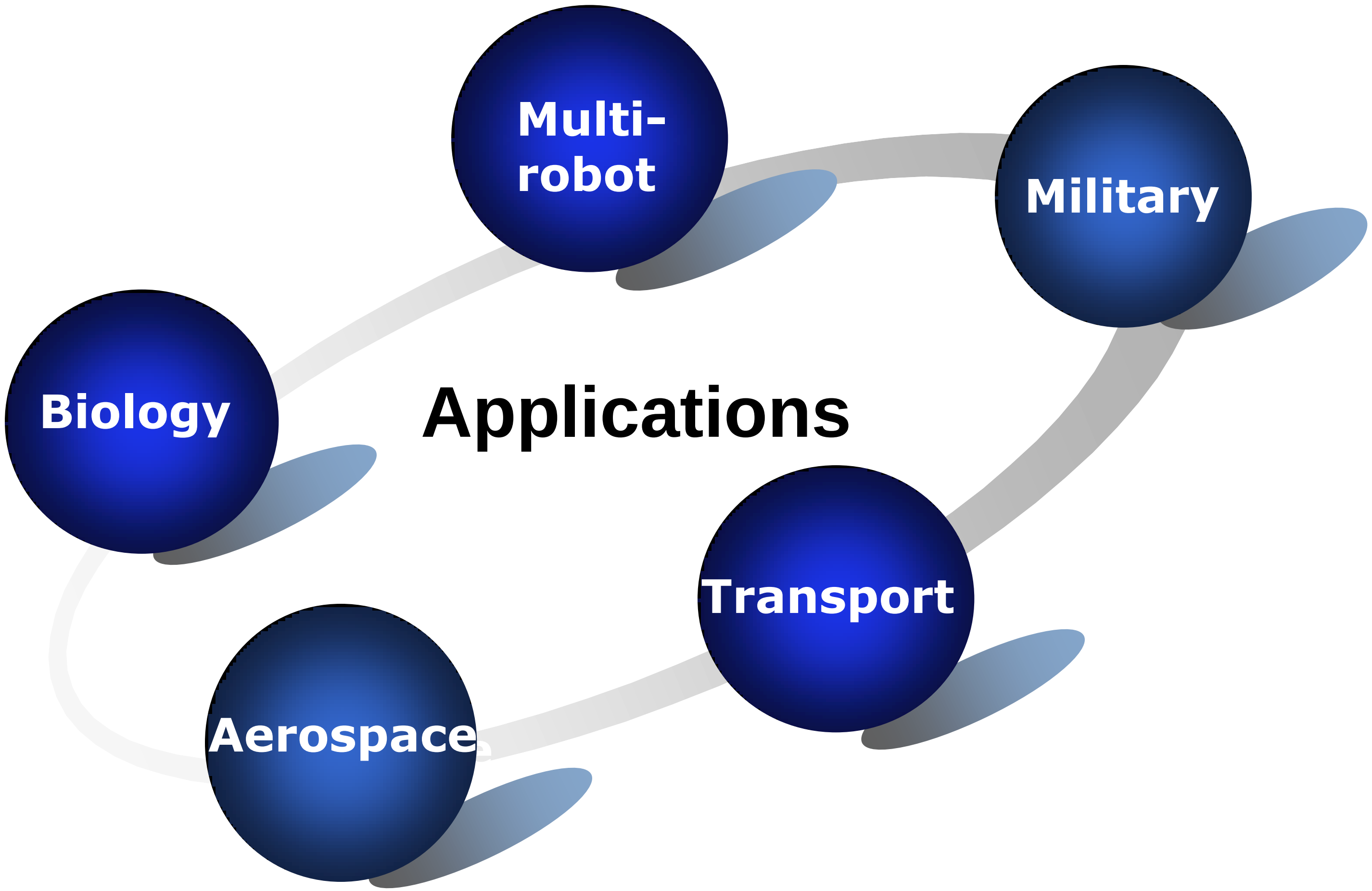
negotiation cooperation

finish tasks

that can not be solved by a single agent

Advantages: flexible, error tolerance, higher efficiency, robustness…

1. Introduction



1. Introduction

Flocking

Biology

Swarming

Foraging

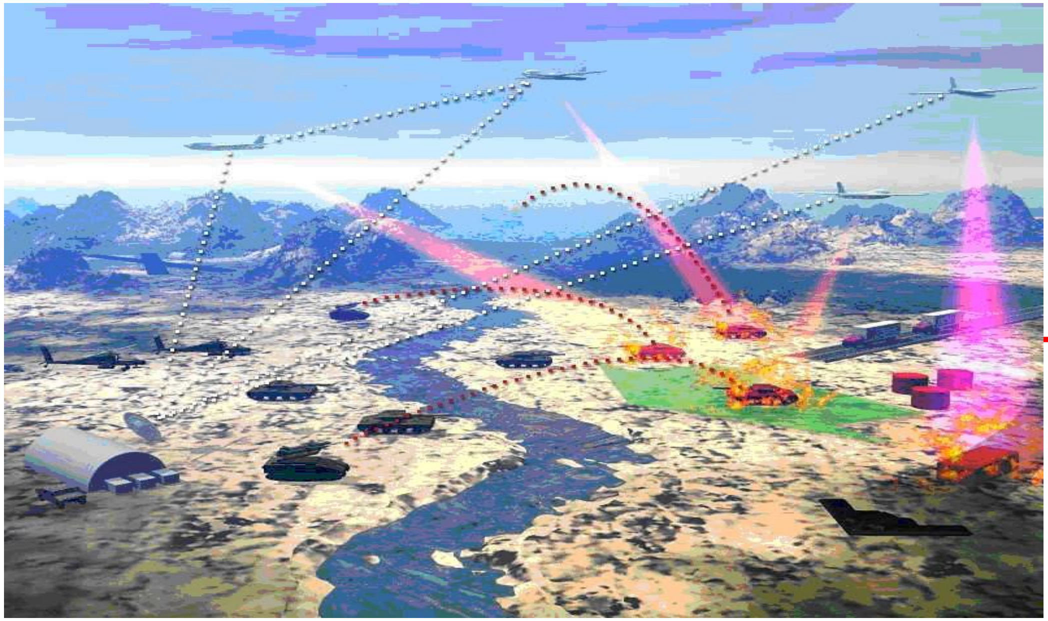
1. Introduction

Multi-robot

Production lines

RoboCup (Robot World Cup)

1. Introduction



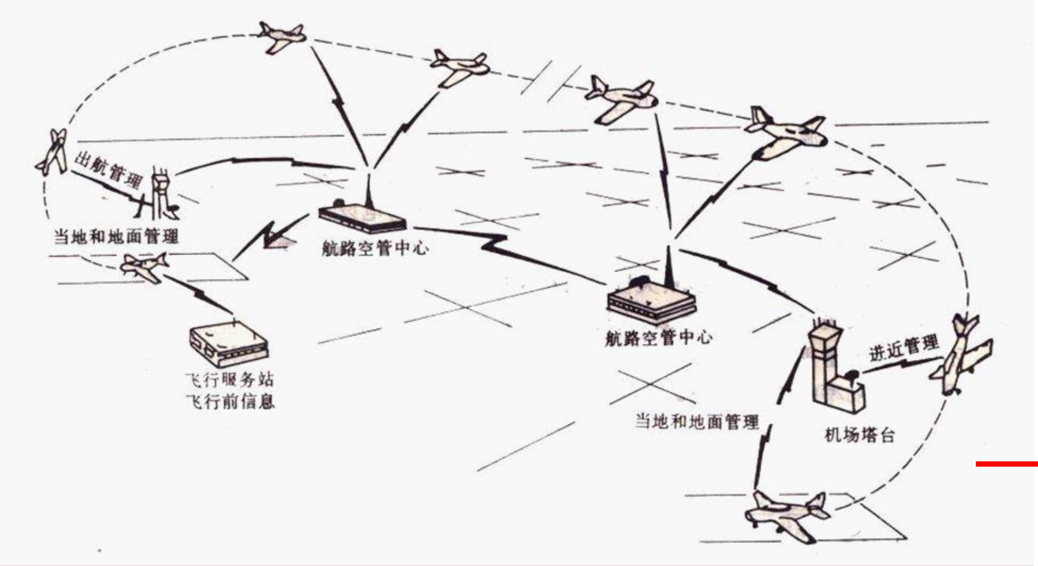
Military

Coordination combat

Formation flight

1. Introduction

Highway systems



Transport

Air traffic control

1. Introduction

TacSat-2 satellite

Constellation of satellites

Aerospace

1. Introduction

Consensus： synchronization, agreement, …

——an essential problem in the field of multi-agent application

Distributed control：

characteristics： centralized management

decentralized control

advantages： flexible, easy management,

high performance-price ratio,

high reliability, …

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3. The output regulation for multi-agent systems
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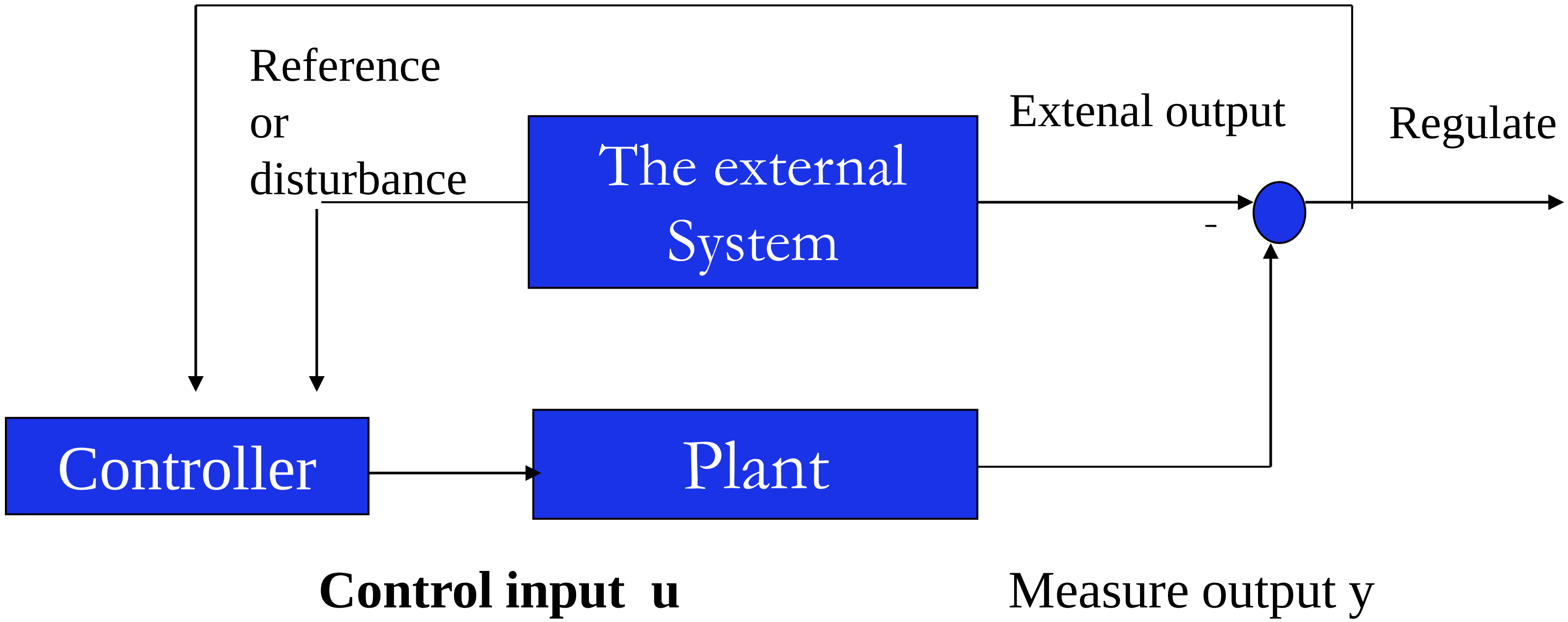
Output regulation is a control design scheme for uncertain system. It aims to control the system states to track reference signals or reject disturbance signals , and keep stable for the closed system.



The brief contents on output regulation theory can be seen in following:

[1] E. J. Davison, “The robust control of a servomechanism problem for linear time-invariant multivariable systems,” IEEE Trans. Automat. Contr., vol. AC-21, pp. 25–34, Jan. 1976.
[2] B. A. Francis, “The linear multivariable regulator problem,” in SIAM J. Control Optim., vol. 15, 1977, pp. 486–505.
[3] B. A. Francis and W. M. Wonham, “The internal model principle of control theory,” Automatica, vol. 12, pp. 457–465, 1976.

Since 1990, Huang J and Isidori A et.al have extended the output regulation theory to nonlinear system.

[4] J. Huang, W. J. Rugh, “On a nonlinear multivariable servomechanism problem,” Automatica, vol. 26, pp. 963–972, 1990.

[5] A. Isidori and C. I. Byrnes, Output regulation of nonlinear systems, IEEE Trans. Automat. Contr., 1990, 35: 131–140.
[6] J Huang, Z Chen.A general framework for tackling the output regulation problem. IEEE Trans. Automatic Control, 2004, 49(12): 2203-2218

Inner Model Principle is developed since 1970 [7], it provide a good solution for the output regulation of linear system.

[7] B. A. Francis, W. M. Wonham. The internal model principle of control theory. Automatica 1976; 12:457-465.

Consider the following linear system:

$$\begin{aligned} \dot{x} &= Ax + Bu + Pw, \\ e &= Cx + Qw, \end{aligned}$$

w **is the state for external system, it satisfy that:**

$$\dot{w} = Sw.$$

Output regulation is to built

$$\dot{\xi} = F\xi + Ge$$
$$u = H\xi$$

To have the closed system

$$\dot{x} = Ax + BH\xi + P\omega$$
$$\dot{\xi} = F\xi + GCx + GQ\omega$$
$$\dot{\omega} = S\omega$$

Satisfy that:

For any initial values：

（I） The system is stable；

（II） $\lim_{t\to\infty} e_i(t) = 0.$

Theorem 1. The necessary and sufficient condition for the existence of output regulation is that（A，B）is stabilization ,（C，A）is detectable，and the linear matrix equation

$$\begin{cases} \Pi S = A\Pi + B\Gamma + P \\ 0 = C\Pi + Q \end{cases}$$

have solution $\Pi, \Gamma.$

Theorem 2. The necessary and sufficient condition for the existence of structure stable output regulation is that（A_0，B_0） is stabilization ,, （C_0，A_0） is detectable, and the and the regulator equation

$$\begin{cases} \Pi S = A_0 \Pi + B_0 \Gamma + P \\ 0 = C_0 \Pi + Q \end{cases}$$

have solutions $\Pi, \Gamma.$

Defination：If the designed regulator for Nominal parameter ｛A_0, B_0, C_0, P_0, Q_0｝ is effective to each ｛A, B, C, P, Q｝ in a neighborhood of it，the regulator is called structure stable.

For nonlinear System

$$\dot{x} = f(x, u, w),$$
$$e = h(x, w),$$

w **is the external system state**

$$\dot{w} = s(w).$$

It is to built output regulator

$$\dot{\xi} = \eta(\xi, e)$$
$$u = \theta(\xi)$$

Satify that the closed system

$$\dot{x} = f(x, \theta(\xi), w)$$
$$\dot{\xi} = \eta(\xi, h(x, w))$$
$$\dot{w} = s(w)$$

For any initial values in the neighbourhood of original：

（I）The system is linear approximate stable；

（II） $$\lim_{t\to\infty} e_i(t) = 0.$$

The internal mode principle also was extended to nonlinear ssystems [5]. The following is the form of nonlinear regulator equation：

$$\begin{cases} \dfrac{\partial \pi}{\partial w} s(w) = f(\pi(w), c(w), w) \\ 0 = h(\pi(w), w) \end{cases}$$

Remark：　The necessary and sufficient condition for the existence of nonlinear output regulation is that the nonlinear output regulation equation canbe solved.

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3 The Output regulation for multi-agent systems

Motivation

Leader-following problem is a *hotspot issue* in coordination control of multi-agent systems.

Usually we assume that:

leader is completely measurable for some follower agents

contradictory

In practice:

Leader may be not completely measurable for every other agents

Output regulation problem for multi-agent systems

The output regulation problem of controlled system is that of controlling a plant to track reference signals or reject disturbance signals.

Motivation

In practice, the leader agents are dynamics, and not all the information is measurable or available in communication. For example, in pursuing and formation control problem, the position and speed states of the leadrs usually constantly change. So it is important to design the distributed control based on distributed estimation for the multi-agent systems.

In recent years, several results about this problem have been given, for example:

Related research

[8] Fax J A, Murray R M, Information flow and cooperative control of vehicle formations, IEEE Trans. on Automatic Control, 49,1453-1476 (2004).

The authors presented distributed control concerning observer design for multi-agent systems, and first tackled this problem.

[9] Hong Y G, Hu J, Gao L, Tracking control for multi-agent consensus with an active leader and variable topology, Automatica, 42(7), 1177-1182 (2006)

A consensus problem for the given multi-agent system with an active leader was considered.

[10] Hong Y G, Chen G, Bushnell L, Distributed observers design for leader-following control of multi-agent networks, Automatica, 44, 846-850 (2008).

The distributed controllers and observers were designed for the second-order follower agents.

3 The Output regulation for multi-agent systems

Related research

[11] Hong Y G, Wang X L, Zhong P, Multi-agent coordination with general linear models: a distributed output regulation approach, Proceedings of the 8th CCA, 137-142 (2010)

The distributed output regulation problem of linear multi-agent systems was presented.

[12] Wang X L, Hong Y G, Huang J, Zhong P, A distributed control approach to robust output regulation of networked linear systems, Proceedings of the 8th CCA, 1853-1857 (2010)

Wang and Hong designed a distributed controller to solve the robust output regulation problem of a networked linear system with uncertainties.

3 The output regulation for multi-agent systems

Our works:

- Consider the multi-agent systems with general noninear dynamics.
- Assume that the exosystem is not measurable completely for other agents.
- Design the distributed feedback controllers by solving the distributed nonlinear output regulation problem and distributed nonlinear robust output regulation problem of multi-agent systems.

Problem Statement

Consider a network system consisting of a leader and N follower agents

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i + h_i(x_i)w,$$ followers

$$\dot{w} = s(w),$$ active leaders (or environmental disturbance)

measured output and reference signal $$y_0 = q(w),$$

$$e_i = x_i - y_0,\ i = 1, 2, \cdots, N,$$

This model will applies to some problem in multi-agent control

- Consensus control for multi-agent with environmental disturbance
- Static leader-following problem
- Active leader-following problem

Distributed output regulation problem

Our control aim is

$$\lim_{t \to +\infty} e_i(t) = 0.$$

Design measurements of the external state received by each agent, and it is related to its neighbors or the leader as follows

$$z_i = \sum_{j \in N_i} a_{ij}(x_i - x_j) + b_i(x_i - y_0).$$

Control Law

(I) distributed static feedback control

$$u_i = \vartheta_i(x_i, z_i).$$

(II) distributed dynamic feedback control

$$u_i = \theta_i(v_i, z_i),$$
$$\dot{v}_i = \eta_i(v_i, z_i).$$

Remark

Compared with the static feedback control, the dynamic feedback control has the better robustness in the output regulation problem.

Distributed output regulation problem

Definition

The distributed output regulation problem of system is solvable with dynamic (static) feedback control, if the following conditions hold:

a) the equilibrium state of the closed-loop system is stable, when $w \equiv 0$.

b) for initial condition $(x(0), v(0), w(0))$, such that

$$\lim_{t \to +\infty} e_i(t) = 0.$$

Distributed output regulation problem

For static feedback control

Theorem

The distributed output regulation problem of considered multi-agent system is solvable with static feedback control law, if and only if there exist $x=\pi(w), u=c(w), \pi(0)=0, c(0)=0$, satisfying

$$\frac{\partial\pi}{\partial w}s(w)= f(\pi(w))+g(\pi(w))c(w)+h(\pi(w))w,$$

$$\pi(w)-\mathbf{1}\otimes q(w)=0.$$

Distributed output regulation problem

Theorem

For dynamic feedback control

The distributed output regulation problem of considered multi-agent system is solvable with dynamic feedback control law, if and only if there exist $x=\pi(w), u=c(w), \pi(0)=0, c(0)=0$, satisfying

$$\frac{\partial \pi}{\partial w}s(w)=f(\pi(w))+g(\pi(w))c(w)+h(\pi(w))w,$$
$$\pi(w)-\mathbf{1}\otimes q(w)=0.$$

such that the following autonomous system with output

$$\begin{aligned}\dot{w}&=s(w),\\ u&=c(w).\end{aligned}$$

is immersed into the system

$$\begin{aligned}\dot{v}&=\vartheta(v),\\ u&=\gamma(v).\end{aligned}$$

and the pairs

$$\begin{pmatrix} A & 0 \\ NH & G_v \end{pmatrix}, \begin{pmatrix} B \\ 0 \end{pmatrix} \quad and \quad \begin{pmatrix} 0 & H \end{pmatrix}, \begin{pmatrix} A & BF_v \\ 0 & G_v \end{pmatrix}$$

is stabilizable and detectable, respectively.

Problem Statement

Consider the network system is modeled as follows

$$
\begin{aligned}
\dot{x}_i &= f_i(x_i) + g_i(x_i)u_i + h_i(x_i)w, \\
\dot{w} &= s(w), \\
y_0 &= q(w), \\
e_i &= x_i - y_0, \ i = 1, 2, \cdots, N,
\end{aligned}
\tag{1}
$$

Problem Statement

Consider the network system is modeled as follows

$$\begin{aligned}
\dot{x}_i &= f_i(x_i) + g_i(x_i)u_i + h_i(x_i)w, \\
\dot{w} &= s(w), \\
y_0 &= q(w), \\
e_i &= x_i - y_0,\ i = 1, 2, \cdots, N,
\end{aligned} \tag{1}$$

where x_i and u_i represent the position state and control input of the i-th agent, respectively. w represents the state of the exosystem $\dot{w} = s(w)$, y_0 is the measured output and reference signal, e_i is the regulated output for the i-th agent, which describes the control target. Assume the functions $f_i(\cdot)$, $g_i(\cdot)$ and $h_i(\cdot)$ are known and smooth with $f_i(0) = 0$, $s(w)$ and $q(w)$ are smooth mappings, with $s(0) = 0$ and $q(0) = 0$.

Problem Statement

Suppose w is not all measurable for every agent, then it cannot be used in the design. Moreover, y_0 may not be available for agent i, so e_i cannot be used directly in its design. Therefore we design the external state measurements received by each agent, and it relative to its neighbors or the leader as follows:

$$z_i = \sum_{j \in N_i} a_{ij}(x_i - x_j) + b_i(x_i - y_0). \tag{2}$$

Define the following dynamic feedback distributed control law:

$$\begin{aligned} u_i &= \theta_i(v_i, z_i), \\ \dot{v}_i &= \eta_i(v_i, z_i), \end{aligned} \tag{3}$$

where $\theta_i(v_i, z_i)$ and $\eta_i(v_i, z_i)$ are $\mathcal{C}^k(k \geq 2)$ mapping. For convenience, we assume $\theta_i(0,0) = 0$ and $\eta_i(0,0) = 0$.

Remark 1.

Different from conventional output regulation, distributed output regulation design is mainly based on the external state measurements z_i of the multi-agent system. In fact, it has to collect the information in a distributed way from its neighbor agents.

Remark 2.

If $h_i(x_i) = 0, i = 1, 2, \cdots, N$, the distributed output regulation problem becomes an active leader-following problem, where the follower agents have to exchange the information of the exosystem (i.e., the active leader) with their neighbors if they can not directly get formation from the leader.

Lemma 1.

Consider the system as follows:

$$\begin{aligned} \dot{y} &= A_1 y + g_1(y, z), \\ \dot{z} &= A_2 z + g_2(y, z), \end{aligned} \tag{4}$$

where $y \in \mathbf{R}^n$, $z \in \mathbf{R}^m$, A_1 and A_2 are constant matrices and all the eigenvalues of A_1 have negative real parts, while all the eigenvalues of A_2 have zero real parts, $g_1(\cdot)$ and $g_2(\cdot)$ are $\mathcal{C}^2$ mappings satisfying

$$g_i(0,0) = 0, \quad \left.\frac{\partial g_i}{\partial y}\right|_{(0,0)} = 0, \quad \left.\frac{\partial g_i}{\partial z}\right|_{(0,0)} = 0.$$

Then there exist a constant $\epsilon > 0$ and a continuously differentiable function $h(z)$ for all $\|z\| < \epsilon$, such that $y = h(z)$ is the center manifold for system (4).

Lemma 2.

Suppose $y = \pi(z)$ is a center manifold for system (4) at $(0, 0)$. Let $(y(t), z(t))$ be a solution curve of (4). There exist a neighborhood $\mathbf{U}^0$ of $(0, 0)$ and real number $\rho > 0$, $\delta > 0$, such that if $(y(0), z(0)) \in \mathbf{U}^0$, then

$$\|y(t) - \pi(z(t))\| \leq \rho e^{-\delta t} \|y(0) - \pi(z(0))\|,$$

for all $t \geq 0$, as long as $(y(t), z(t)) \in \mathbf{U}^0$.

Lemma 3.

Suppose the system

$$\dot{x} = Ax + Bu, \tag{5}$$
$$y = Cx, \tag{6}$$

is stabilizable and detectable, then the system is stabilizable by the dynamic output feedback

$$u = M\hat{x}, \tag{7}$$
$$\dot{\hat{x}} = K\hat{x} + Ey, \tag{8}$$

where M, K and E are the appropriate matrices.

Substituting (6) and (7) into (8) and (5), respectively, yields

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & BM \\ EC & K \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}. \tag{9}$$

By Lemma 3, the eigenvalues of the matrix

$$\begin{pmatrix} A & BM \\ EC & K \end{pmatrix}$$

are all with negative real parts

Definition 1.

(Poisson stable) In a nonlinear system $\dot{w} = s(w)$, an initial condition w_0 is said to be Poisson stable if the flow $\Phi_t^s(w_0)$ of the vector field $s(w)$ is defined for all $t \in \mathbf{R}$, and for each neighborhood U of w_0 and for each real number $T > 0$, there a time $t_1 > T$ such that $\Phi_{t_1}^s(w_0) \in U$, and a time $t_2 < -T$ such that $\Phi_{t_2}^s(w_0) \in U$.

Definition 2.

Consider the two following dynamic systems

$$\begin{aligned} \dot{x} &= f(x), \ x \in \mathbf{X}, \\ y &= h(x), \ z \in \mathbf{R}^m, \end{aligned} \tag{10}$$

and

$$\begin{aligned} \dot{z} &= g(z), \ z \in \mathbf{Z}, \\ y &= l(z). \ y \in \mathbf{R}^m, \end{aligned} \tag{11}$$

Then the system (10) is immersed into the system (11) if there exists a smooth mapping $\tau : \mathbf{X} \to \mathbf{Z}$, satisfying $\tau(0) = 0$, such that

$$\begin{aligned} \frac{\partial \tau}{\partial x} f(x) &= g(\tau(x)), \\ h(x) &= l(\tau(x)), \end{aligned} \tag{12}$$

for all $x \in \mathbf{X}$.

Denote

$$x = \begin{pmatrix} x_1 & x_2 & \cdots & x_N \end{pmatrix}^T, \quad u = \begin{pmatrix} u_1 & u_2 & \cdots & u_N \end{pmatrix}^T,$$

$$e = \begin{pmatrix} e_1 & e_2 & \cdots & e_N \end{pmatrix}^T, \quad z = \begin{pmatrix} z_1 & z_2 & \cdots & z_N \end{pmatrix}^T,$$

$$f(x) = \begin{pmatrix} f_1(x_1) & f_2(x_2) & \cdots & f_N(x_N) \end{pmatrix}^T,$$

$$h(x) = \begin{pmatrix} h_1(x_1) & h_2(x_2) & \cdots & h_N(x_N) \end{pmatrix}^T,$$

$$g(x) = diag\left(g_1(x_1), \ g_2(x_2), \ \cdots, \ g_N(x_N) \right),$$

$$\theta(v, z) = \begin{pmatrix} \theta_1(v_1, z_1) & \theta_2(v_2, z_2) & \cdots & \theta_N(v_N, z_N) \end{pmatrix}^T,$$

$$\eta(v, z) = \begin{pmatrix} \eta_1(v_1, z_1) & \eta_2(v_2, z_2) & \cdots & \eta_N(v_N, z_N) \end{pmatrix}^T.$$

Then system (1) and the controller (3) can be rewritten as

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)u + h(x)w, \\
\dot{w} &= s(w), \\
e &= x - \mathbf{1} \otimes q(w)
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
u &= \theta(v, z), \\
\dot{v} &= \eta(v, z).
\end{aligned}
\tag{14}
$$

Let $B_0 = diag(b_1, b_2, \cdots, b_N)$, the external state measurements (2) can also be rewritten as

$$
z = Hx - (B_0\mathbf{1}) \otimes y_0, \tag{15}
$$

where $H = L + B_0$.

Substituting (14) into (13) yields a closed-loop system

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)\theta(v, z) + h(x)w, \\
\dot{v} &= \eta(v, z), \\
\dot{w} &= s(w), \\
e &= x - \mathbf{1} \otimes q(w).
\end{aligned}
\tag{16}
$$

Then we can obtain $f(0) = 0$, $s(0) = 0$ and $q(0) = 0$. For $\theta(0, 0) = 0$ and $\eta(0, 0) = 0$, the closed-loop system (16) has an equilibrium state $(x, v, w) = (0, 0, 0)$.

Definition 3.

The distributed output regulation problem of system (16) is solvable, if the following conditions hold

a) the equilibrium state $(x, v) = (0, 0)$ of system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\theta(v, z) + h(x)w, \\ \dot{v} &= \eta(v, z) \end{aligned} \tag{17}$$

is stable when $w \equiv 0$.

b) for any initial condition $(x(0), v(0), w(0))$, such that

$$\lim_{t \to +\infty} e_i(t) = 0.$$

Now we give the following assumptions for solving the distributed output regulation problem.

- (A1) The leader is globally reachable in $\bar{\mathcal{G}}$.

- (A2) The point $w = 0$ is a stable equilibrium of $\dot{w} = s(w)$, and each initial condition $w(0)$ is Poisson stable.

According to the Taylor expansion, we can obtain the linear approximation of the nonlinear system (13). At the equilibrium $(x, w) = (0, 0)$, this approximation system has the following form

$$\begin{aligned} \dot{x} &= Ax + Bu + Cw, \\ \dot{w} &= Sw, \\ e &= x - (\mathbf{1} \otimes Q)w, \end{aligned} \tag{18}$$

where

$$A = \left[\frac{\partial f}{\partial x}\right]_{x=0}, \quad B = g(0), \quad C = h(0),$$
$$S = \left[\frac{\partial s}{\partial w}\right]_{w=0}, \quad Q = \left[\frac{\partial q}{\partial w}\right]_{w=0}.$$

Similarly, the distributed dynamic feedback control law (14) can be approximated as

$$\begin{aligned} u &= F_v v + F_z z, \\ \dot{v} &= G_v v + G_z z \end{aligned} \tag{19}$$

with

$$F_v = \left[\frac{\partial\theta}{\partial v}\right]_{v=0,z=0}, \quad F_z = \left[\frac{\partial\theta}{\partial z}\right]_{v=0,z=0},$$
$$G_v = \left[\frac{\partial\eta}{\partial v}\right]_{v=0,z=0}, \quad G_z = \left[\frac{\partial\eta}{\partial z}\right]_{v=0,z=0}.$$

Next we will consider the distributed output regulation problem for the multi-agent system (1). To begin with, we give the following result.

Theorem 1.

For hypothesis A2, assume that a) in Definition 3 is fulfilled, then the following statements are equivalent
i) b) in Definition 3 is fulfilled;
ii) there exist $\mathcal{C}_k (k \geq 2)$ mapping $x = \pi(w)$ with $\pi(0) = 0$, and $v = \sigma(w)$ with $\sigma(0) = 0$, such that

$$\frac{\partial \pi}{\partial w} s(w) = f(\pi(w)) + g(\pi(w))\theta(\sigma(w), 0) + p(\pi(w))w, \quad (20)$$

$$\frac{\partial \sigma}{\partial w} s(w) = \eta(\sigma(w), 0), \quad (21)$$

$$\pi(w) - \mathbf{1} \otimes q(w) = 0. \quad (22)$$

Proof. i)$\to$ii)

Using the Taylor expansion, the closed-loop system (16) can be written as

$$\begin{aligned} \dot{x} &= (A + BF_z H)x + BF_v v - (BF_z B_0 \mathbf{1}) \otimes Qw + Cw + \phi(x, v, w), \\ \dot{v} &= G_z Hx + G_v v - (G_z B_0 \mathbf{1}) \otimes Qw + \varphi(x, v, w), \\ \dot{w} &= Sw + \psi(w), \end{aligned} \tag{23}$$

where $\phi(\cdot)$, $\varphi(\cdot)$ and $\psi(\cdot)$ are $\mathcal{C}^2$ mappings with $\phi(0,0,0) = 0$, $D\phi(0,0,0) = 0$, $\varphi(0,0,0) = 0$, $D\varphi(0,0,0) = 0$, $\psi(0) = 0$ and $D\psi(0) = 0$.

i)→ii)

From the assumption, a) is fulfilled, then the eigenvalues of the matrix

$$\begin{pmatrix} A + BF_z H & BF_v \\ G_z H & G_v \end{pmatrix}$$

are all with negative real parts. Otherwise, by A2, we know the eigenvalues of the matrix S are on the imaginary axis. Therefore, from Lemma 1, there exist continuously differentiable functions $\pi(w)$ and $\sigma(w)$, such that

$$x = \pi(w)$$
$$v = \sigma(w)$$

is the center manifold at $(0, 0, 0)$ of the system (23).

i)→ii)

Substitute $x = \pi(w), v = \sigma(w)$ into the first two equations of (23), we have

$$\frac{\partial \pi}{\partial w} s(w) = f(\pi(w)) + g(\pi(w))\theta(\sigma(w), z(w)) + p(\pi(w))w,$$

$$\frac{\partial \sigma}{\partial w} s(w) = \eta(\sigma(w), z(w)),$$

with

$$z(w) = H\pi(w) - (B_0\mathbf{1}) \otimes q(w).$$

i)→ii)

Suppose w_0 is any initial condition of $\dot{w} = s(w)$, the trajectory $(x(t), v(t), w(t))$ of system (16) satisfies $(x(0), v(0), w(0)) = (\pi(w_0), \sigma(w_0), w_0)$. In light of A2, for any $\epsilon > 0$ and every $T > 0$, there exist some $t > T$, such that

$$\|(x(t), v(t), w(t)) - (\pi(w_0), \sigma(w_0), w_0)\| < \epsilon.$$

So every trajectory on the center manifold can not converge to zero. Thus b) is fulfilled only if

$$e = x(t) - \mathbf{1} \otimes q(w(t)) = 0$$

at $t = 0$, i.e., (22) holds. Therefore, we have $\pi_i(w) = x_i = q(w)$. Then from the external state measurements form (2), we can obviously see that $z(w) = 0$, so the equation (20) and (21) hold.

ii)→i)

From the condition (22), we know that

$$\begin{aligned} e(t) &= x(t) - \mathbf{1} \otimes q(w(t)) - [\pi(w(t)) - \mathbf{1} \otimes q(w(t))] \\ &= x(t) - \pi(w(t)). \end{aligned} \tag{24}$$

On the other hand, (20) is satisfied, the mapping

$$x = \pi(w)$$
$$v = \sigma(w)$$

with $\pi(0) = 0, \sigma(0) = 0$ is a center manifold for system (16).

ii)→i)

Then from Lemma 2, for any $\rho > 0,\ \delta > 0$ and all $x(0),\ w(0)$ which sufficiently close to the equilibrium state $(x, w) = (0, 0)$, we have

$$\|x(t) - \pi(w(t))\| \leq \rho e^{-\delta t}\|x(0) - \pi(w(0))\|.$$

Thus

$$\lim_{t\to\infty}[x(t) - \pi(w(t))] = 0,$$

i.e., $\lim_{t\to\infty} e(t) = 0$, so the condition b) is satisfied. The proof is completed here.

Theorem 2.

Under A1,A2, the distributed output regulation problem of system (1) is solvable with a appropriate control law, if and only if there exist $x = \pi(w)$, $u = c(w)$ with $\pi(0) = 0$, $c(0) = 0$ satisfying

$$\begin{aligned} \frac{\partial \pi}{\partial w} s(w) = f(\pi(w)) + g(\pi(w))c(w) + p(\pi(w))w, \\ \pi(w) - \mathbf{1} \otimes q(w) = 0. \end{aligned} \tag{25}$$

such that the following autonomous system with output

$$\begin{aligned} \dot{w} &= s(w), \\ u &= c(w). \end{aligned} \tag{26}$$

is immersed into the system

$$\begin{aligned} \dot{v} &= \vartheta(v), \\ u &= \gamma(v). \end{aligned} \tag{27}$$

where $\vartheta(0) = 0, \gamma(0) = 0$, for some choice of the matrix N and the matrices

$$G_v = \left[\frac{\partial \vartheta}{\partial v}\right]_{v=0}, \quad F_v = \left[\frac{\partial \gamma}{\partial v}\right]_{v=0},$$

such that the pair

$$\begin{pmatrix} A & 0 \\ NH & G_v \end{pmatrix}, \begin{pmatrix} B \\ 0 \end{pmatrix} \tag{28}$$

is stabilizable and the pair

$$\begin{pmatrix} 0 & H \end{pmatrix}, \begin{pmatrix} A & BF_v \\ 0 & G_v \end{pmatrix} \tag{29}$$

is detectable.

Proof: necessity

Suppose the distributed output regulation problem of system (1) is solvable for the distributed dynamic feedback controller (14).

By Theorem 1, there exist mappings $x = \pi(w)$ with $\pi(0) = 0$ and $v = \sigma(w)$ with $\sigma(0) = 0$, such that the equations (20)-(22) hold. Set $c(w) = \theta(\sigma(w), 0)$, and substitute it into (20), then the condition (25) is satisfied for $\pi(w)$ and $c(w)$.

Necessity

Let $\gamma(v) = \theta(v, 0)$, $\vartheta(v) = \eta(v, 0)$, observe that $\gamma(v)$ and $\vartheta(v)$ satisfy

$$\begin{aligned} \frac{\partial \sigma}{\partial w} s(w) &= \vartheta(\sigma(w)), \\ c(w) &= \gamma(\sigma(w)). \end{aligned} \tag{30}$$

This goes to show that the autonomous system (26) is immersed into the system (27), where $v = \sigma(w)$ with $\sigma(0) = 0$. Then we have

$$\left[\frac{\partial \eta}{\partial v}\right]_{v=0,z=0} = \left[\frac{\partial \vartheta}{\partial v}\right]_{v=0} = G_v, \quad \left[\frac{\partial \theta}{\partial v}\right]_{v=0,z=0} = \left[\frac{\partial \gamma}{\partial v}\right]_{v=0} = F_v.$$

Necessity

According to our hypothesis, the distributed output regulation problem is solvable, therefore the eigenvalues of the matrix

$$\begin{pmatrix} A + BF_z H & BF_v \\ G_z H & G_v \end{pmatrix}$$

are all with negative real parts. Observe that

$$\begin{aligned} \begin{pmatrix} A + BF_z H & BF_v \\ G_z H & G_v \end{pmatrix} &= \begin{pmatrix} A & 0 \\ G_z H & G_v \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} \begin{pmatrix} F_z H & F_v \end{pmatrix}, \\ &= \begin{pmatrix} A & BF_v \\ 0 & G_v \end{pmatrix} + \begin{pmatrix} BF_z \\ G_z \end{pmatrix} \begin{pmatrix} H & 0 \end{pmatrix}, \end{aligned}$$

so the pair (28) is stabilizable, when $N = G_z$, and the pair (29) is detectable.

Sufficiency

By the conditions, the pair (28) is stabilizable for the chosen matrix N, and the pair (29) is detectable, so that the pair

$$\begin{pmatrix} A + BF_z H & BF_v \\ NH & G_v \end{pmatrix}, \begin{pmatrix} B \\ 0 \end{pmatrix}$$

is also stabilizable, and the pair

$$\begin{pmatrix} H & 0 \end{pmatrix}, \begin{pmatrix} A + BF_z H & BF_v \\ NH & G_v \end{pmatrix}$$

is detectable for the matrix F_z.

Sufficiency

Then by Lemma 3, there exist the matrices M, K, E, such that the matrix

$$\Psi = \begin{pmatrix} \begin{pmatrix} A+BF_zH & BF_v \\ NH & G_v \end{pmatrix} & \begin{pmatrix} B \\ 0 \end{pmatrix} M \\ E\begin{pmatrix} H & 0 \end{pmatrix} & K \end{pmatrix} = \begin{pmatrix} A+BF_zH & BF_v & BM \\ NH & G_v & 0 \\ EH & 0 & K \end{pmatrix} \tag{31}$$

has all eigenvalues with negative real part.

Sufficiency

Now design the controller as follows

$$\begin{aligned} u &= Mv_0 + \gamma(v_1) + F_z z, \\ \dot{v}_0 &= Kv_0 + Ez, \\ \dot{v}_1 &= \vartheta(v_0) + Nz. \end{aligned} \tag{32}$$

Combining (32), (15) and (17), when $w = 0$, we have

$$\begin{aligned} \dot{x} &= f(x) + g(x)[Mv_0 + \gamma(v_1) + F_z Hx], \\ \dot{v}_0 &= Kv_0 + EHx, \\ \dot{v}_1 &= \vartheta(v_0) + NHx. \end{aligned} \tag{33}$$

Sufficiency

It is easy to see that the Jacobian matrix of the system (33) can be written as

$$\Psi' = \begin{pmatrix} A + BF_z H & BM & BF_v \\ EH & K & 0 \\ NH & 0 & G_v \end{pmatrix} \tag{34}$$

at $(x, v_0, v_1) = (0, 0, 0)$. From the matrix Ψ in (31) has all eigenvalues with negative real part, the eigenvalues of the matrix (34) also have negative real part. Therefore, the requirement a) in Definition 3 is satisfied for the designed controller (32).

Sufficiency

Furthermore, there exist mappings $x = \pi(w)$ and $u = c(w)$ with $\pi(0) = 0$ and $c(0) = 0$, such that (25) holds, and there exists $v_1 = \tau(w)$ satisfying

$$\begin{aligned} \frac{\partial \tau}{\partial w} s(w) &= \vartheta(\tau(w)), \\ c(w) &= \gamma(\tau(w)). \end{aligned} \tag{35}$$

Let $\begin{pmatrix} v_0 \\ v_1 \end{pmatrix} = \sigma(w) = \begin{pmatrix} 0 \\ \tau(w) \end{pmatrix}$, one can see that the equations (20)-(22) hold. Then, by Theorem 1, the requirement b) in Definition 3 is also fulfilled. So it concludes that the distributed output regulation problem of system (1) is solvable. This completes the proof.

Remark 3.

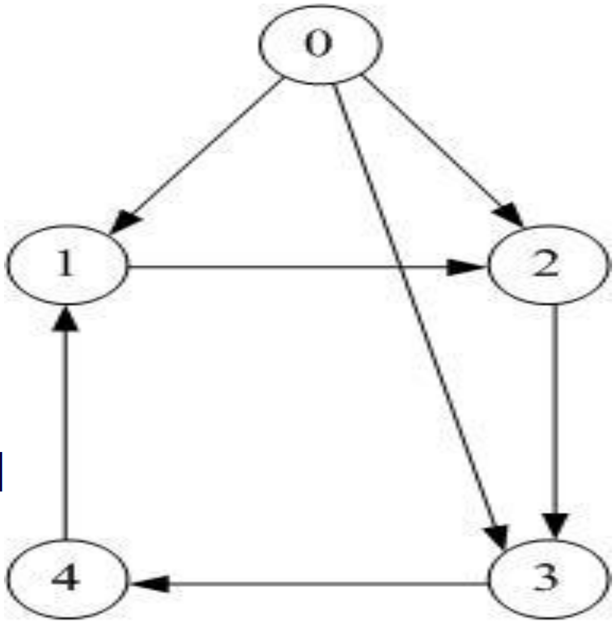
It should be pointed out that the network type of the multi-agent system considered in Theorem 2 is without restrictions. Namely, Theorem 2 can be suitable to directed networks as well as undirected networks of the multi-agent systems.

Remark 4.

Compared with the static feedback control, the dynamic feedback control has the better robustness in the output regulation problem. Then the analysis and design of the distributed dynamic feedback control receive more attention.

Consider a multi-agent system consisting of four following agents with directed graph described by the Laplacian:

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$



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and the diagonal matrix for the interconnection between the l and the following agents is

$$B_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The dynamics of the following agents is described as

$$\dot{x}_i(t) = u_i(t), \ \ i = 1, 2, 3, 4, \tag{36}$$

and an active leader is modeled as follows

$$\begin{aligned} \dot{w}_1(t) &= w_2(t), \\ \dot{w}_2(t) &= -w_1(t), \\ y_0 &= w_1. \end{aligned} \tag{37}$$

Suppose the regulated output is

$$e_i(t) = x_i(t) - y_0.$$

Our control target is to design the distributed dynamic feedback controller as the form of (32), so that $\lim_{t \to +\infty} e_i(t) = 0, \ \ i = 1, 2, 3, 4.$

Furthermore, the autonomous system with outputs (26) can be immersed into the following system

$$\dot{v} = \vartheta(v) = \begin{pmatrix} G_{v1} & 0 & 0 & 0 \\ 0 & G_{v2} & 0 & 0 \\ 0 & 0 & G_{v3} & 0 \\ 0 & 0 & 0 & G_{v4} \end{pmatrix} v = G_v v,$$

$$u = \gamma(v) = \begin{pmatrix} F_{v1} & 0 & 0 & 0 \\ 0 & F_{v2} & 0 & 0 \\ 0 & 0 & F_{v3} & 0 \\ 0 & 0 & 0 & F_{v4} \end{pmatrix} v = F_v v,$$

where

$$G_{vi} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad F_{vi} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3, 4.$$

Then we choose matrices $K = -I$, $E = -I$, $M = I$, and

$$F_z = \begin{bmatrix} -1.8774 & -0.6256 & -0.0050 & -0.1953 \\ 0.0122 & -1.8655 & -0.6018 & -0.4962 \\ -0.5900 & 0.0359 & -1.7955 & -1.0980 \\ 0.1056 & 0.1056 & 0.4016 & -2.4023 \end{bmatrix},$$

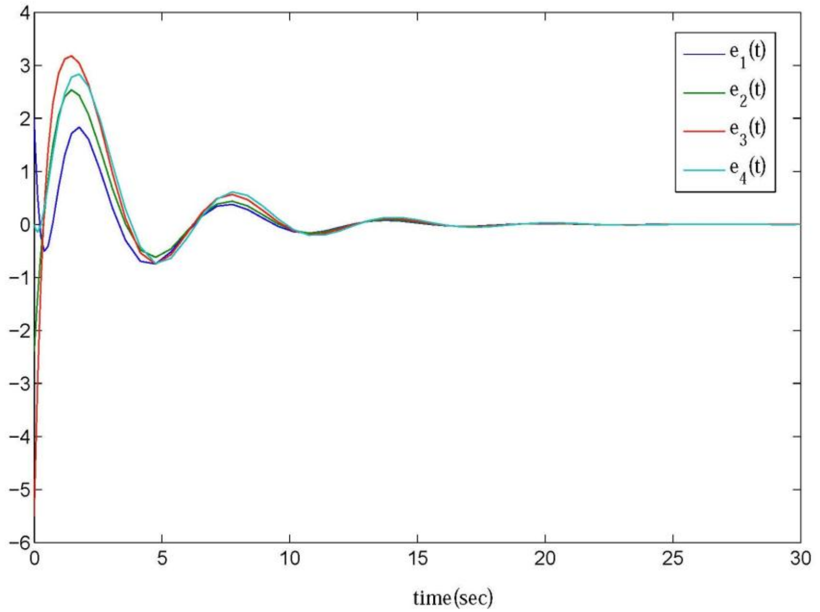
$$N = \begin{bmatrix} -2.3321 & -0.5899 & 0.2700 & -0.1014 \\ -0.9282 & -0.0977 & 0.3087 & 0.0455 \\ 0.9689 & 0.2958 & -0.0380 & 0.0615 \\ 0.3002 & -2.3131 & -0.5519 & -0.3773 \\ 0.3261 & -0.9197 & -0.0807 & 0.0052 \\ -0.0498 & 0.9600 & 0.2778 & 0.2004 \\ -0.5329 & 0.3382 & -2.1955 & -0.9292 \\ -0.0721 & 0.3431 & -0.8594 & -0.0755 \\ 0.2689 & -0.0677 & 0.9095 & 0.4783 \\ 0.1746 & 0.1746 & 0.7204 & -2.4774 \\ 0.0858 & 0.0858 & 0.4349 & -0.6313 \\ -0.0774 & -0.0774 & -0.2543 & 1.1493 \end{bmatrix},$$

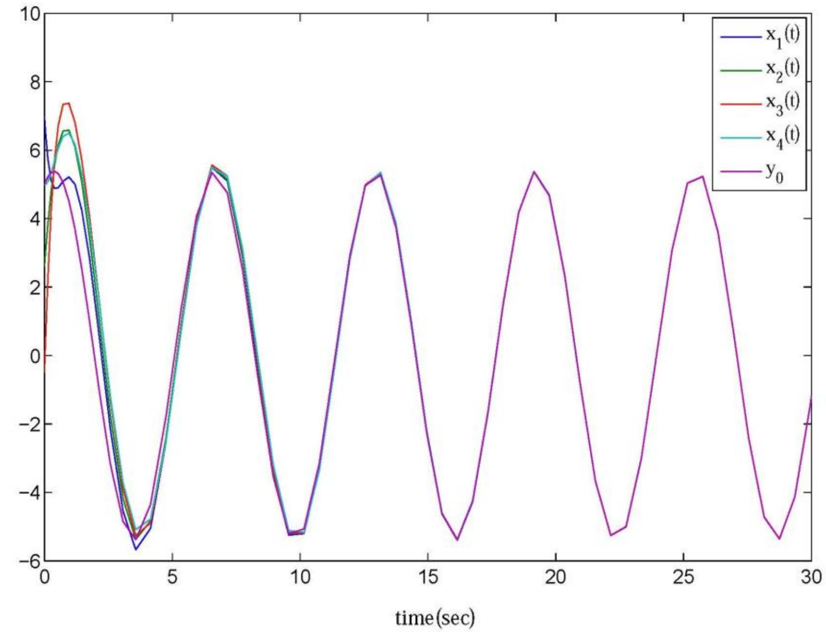
Therefore, by Theorem 2, the output regulation problem of the system in this example is solved. With the initial conditions

$$x_1(0) = 7, \quad x_2(0) = 2.6, \quad x_3(0) = -0.5, \quad x_4(0) = 5,$$
$$w_1(0) = 5, \quad w_2(0) = 2,$$

the trajectories of regulated outputs for the four following agents is shown in Figure 1, and Figure 2 describes the position tracking behaviors of the followings.

Distributed output regulation problem





The trajectories of regulated outputs and position tracking for the four followers.

Outline

1. Introduction
2. The introduction for output regulation theory
3. The output regulation for multi-agent systems
4. The robust output regulation for multi-agent systems
5. Conclusions

Problem Statement

Consider a network system as follows

$$\dot{x}_i = f_i(x_i, u_i, w, \mu),$$
$$\dot{w} = s(w),$$
$$y_0 = q(w),$$
$$e_i = x_i - y_0, \quad i = 1, 2, \cdots, N,$$

uncertainty parameters

Our control aim is

$$\lim_{t \to +\infty} e_i(t) = 0, \quad i = 1, 2, \cdots, N.$$

Distributed robust output regulation problem

Define a virtual regulated output for the i-th agent

$$e_{vi} = \sum_{j \in N_i} a_{ij}(e_i - e_j) + b_i(x_i - y_0), \quad i = 1, 2, \cdots, N.$$

and design the distributed dynamic feedback control law

$$\begin{aligned} u_i &= \theta_i(z_i, e_{vi}), \\ \dot{z}_i &= \eta_i(z_i, e_{vi}). \end{aligned}$$

Definition

For a uncertainty parameter $\mu \in D \in \mathbb{R}^p$, the distributed robust output regulation problem of system is solvable with dynamic feedback control, if the following conditions hold:

a) the equilibrium state of the closed-loop system is robust stable, when $w \equiv 0$.

b) for initial condition $(x_i(0), z_i(0), w(0))$, such that

$$\lim_{t \to +\infty} e_i(t) = 0.$$

Distributed robust output regulation problem

Theorem

For a uncertainty parameter $\mu \in D \in \mathbb{R}^p$, the distributed robust output regulation problem of considered multi-agent system is solvable, if and only if there exist $x = \pi^\mu(w,\mu), u = c(w)$, satisfying

$$\frac{\partial \pi^\mu(w,\mu)}{\partial w} s(w) = f(\pi^\mu(w,\mu), c^\mu(w,\mu), w, \mu),$$
$$\pi(w) - \mathbf{1} \otimes q(w) = 0.$$

such that the following autonomous system with output

$$\dot{w}^\mu = s^\mu(w^\mu), \quad u = c(w).$$

is immersed into the system

$$\dot{z} = \varphi(z), \quad u = \gamma(z).$$

and the pairs

$$\begin{pmatrix} A(\mu) & 0 \\ NH & G_z \end{pmatrix}, \begin{pmatrix} B(\mu) \\ 0 \end{pmatrix} \quad and \quad \begin{pmatrix} 0 & H \end{pmatrix}, \begin{pmatrix} A(\mu) & B(\mu)F_z \\ 0 & G_z \end{pmatrix}$$

is stabilizable and detectable, respectively.

An illustrative example

Example. Consider a nonlinear multi-agent system with the following parameters:

$$L=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & -1 & 0\\ -1 & 0 & 1 & 0\\ 0 & 0 & -1 & 1\end{bmatrix},\quad B_0=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},\quad \mu=0.02.$$

an active leader is modeled as

$$\dot{w}_1(t)=w_2(t),$$
$$\dot{w}_2(t)=-w_1(t).$$

Distributed robust output regulation problem

The follower agents take the dynamic forms as:

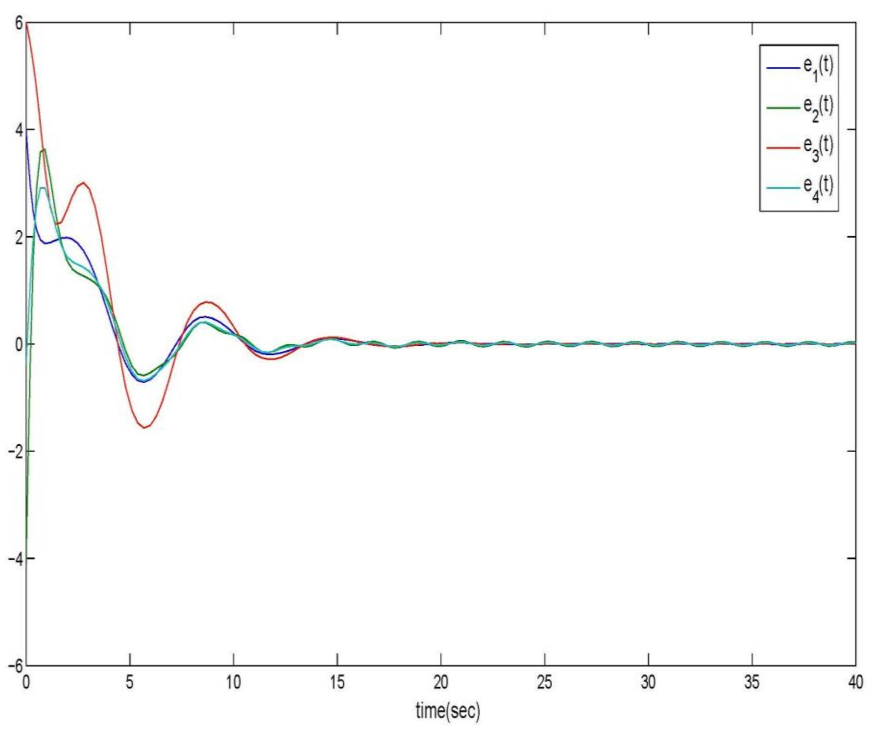
$$\begin{aligned}
\dot{x}_1(t) &= u_1(t), \\
\dot{x}_2(t) &= \mu x_2(t) + u_2(t), \\
\dot{x}_3(t) &= \mu x_3^2 + u_3(t), \\
\dot{x}_4(t) &= u_4(t),
\end{aligned}$$

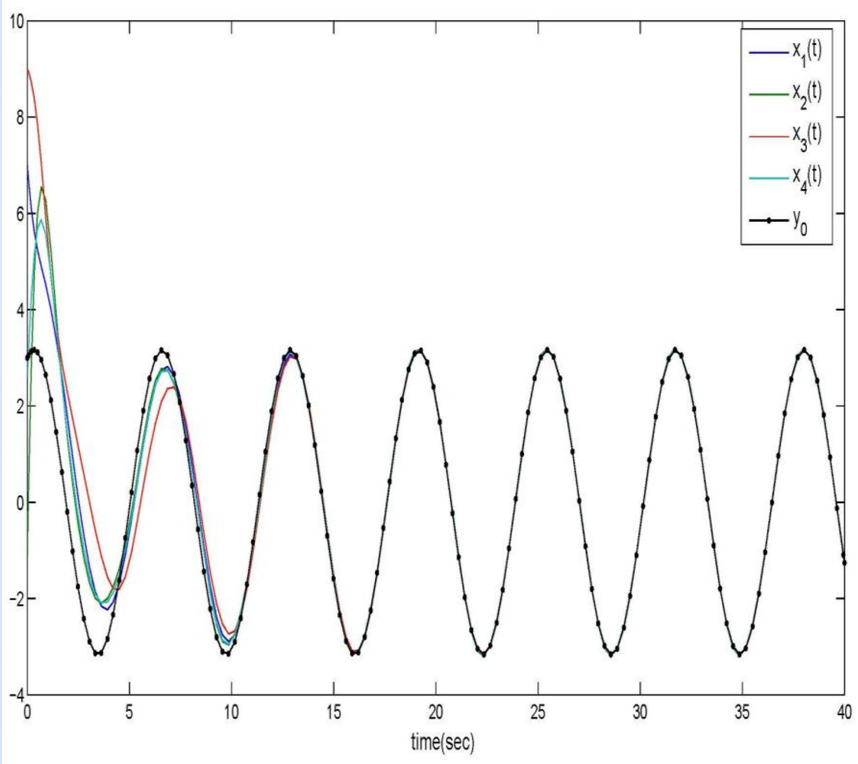
Our control target is

$$\lim_{t \to +\infty} e_i(t) = 0, \quad i = 1, 2, 3, 4.$$

with the dynamic feedback control.

Distributed robust output regulation problem





The trajectories of regulated outputs and position tracking for the four followers.

Outline

1. Introduction
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5 Conclusions

- Studied the problem of distributed output regulation and robust output regulation for the multi-agent systems with general nonlinear dynamic
- Suppose the exosystem (the leader or environment disturbance) can not be measurable completely for other agents
- Designed the distributed feedback control to make the considered multi-agent systems to track the reference or reject disturbance

Publications

1. **Jia Liu**, Zhongxin Liu, Zengqiang Chen. Coordinative control of multi-agent systems using distributed nonlinear output regulation. Nonlinear Dynamics, 67(3): 1871-1881, 2012.
2. **Jia Liu**, Zhongxin Liu, Zengqiang Chen. Coordinative control on output regulation of multi-agent systems with general nonlinear dynamics. Proceedings of the 30th Chinese Control Conference, Yantai, China, July 22-24, 2011, 6007-6011.

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Thank You !