A Unified Framework for Synchronization in Complex Networks and Consensus in Multi-agent Systems

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Outline

- **Background**

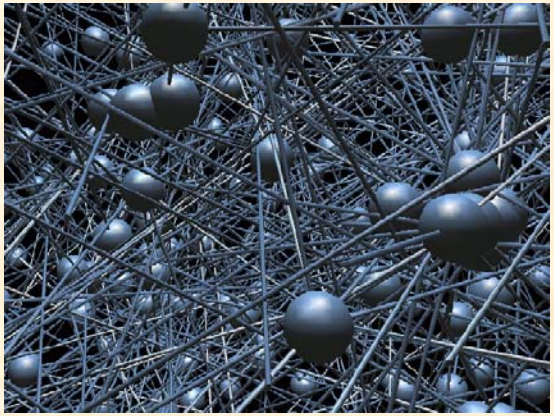
- **Synchronization in Complex Networks and Consensus in Multi-agent Systems**

- **A Unified Framework**

- **Conclusions**

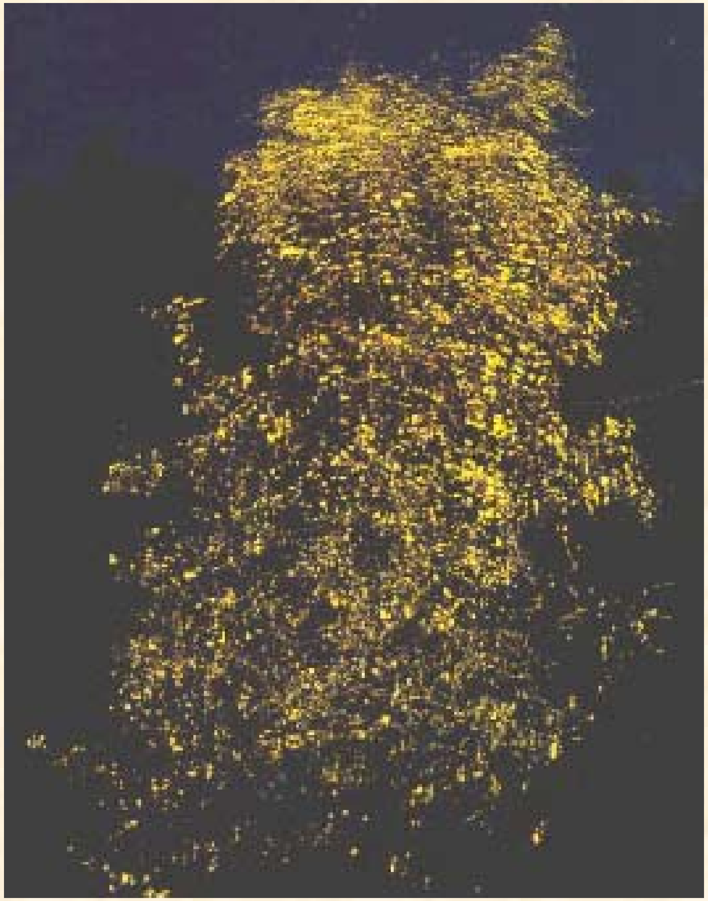
Y. Cao, **W. Yu**, W. Ren, G. Chen, An overview of recent results in distributed multi-vehicle coordination, Submitted to IEEE Trans. Industrial Informatics.

Background: Network Topology

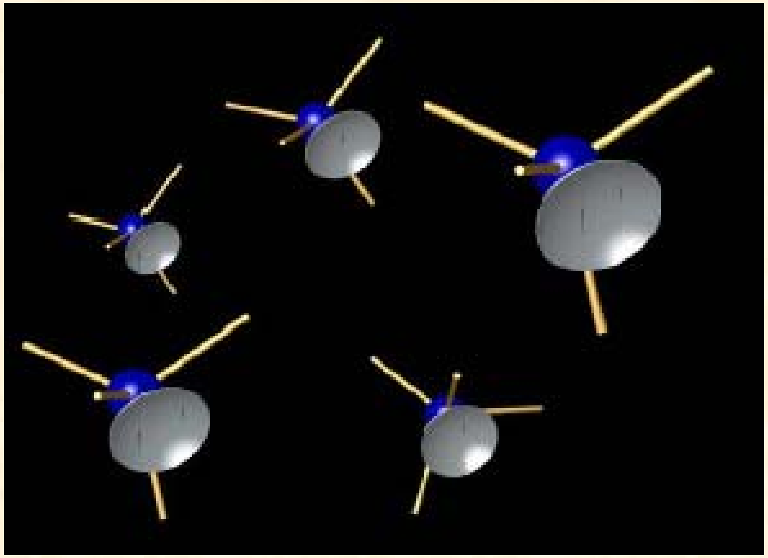


- A network is a set of nodes interconnected via links
  - Internet: Nodes – routers Links – wires
  - Neural Network: Nodes – cells Links – nerves
  - Social Networks: Nodes – individuals Links – relations
  - … …

Fireflies Synchronization



Attitude Alignment



The attitude of each spacecraft is synchronized with its two adjacent neighbors via a bi-directional communication channel

Fish Swarming

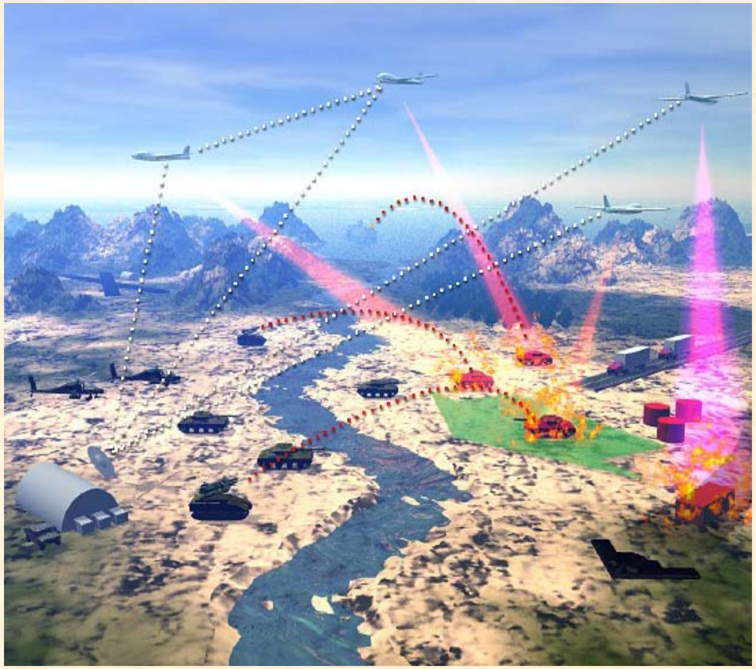
- Swarming:
  to move
  or gather
  in group

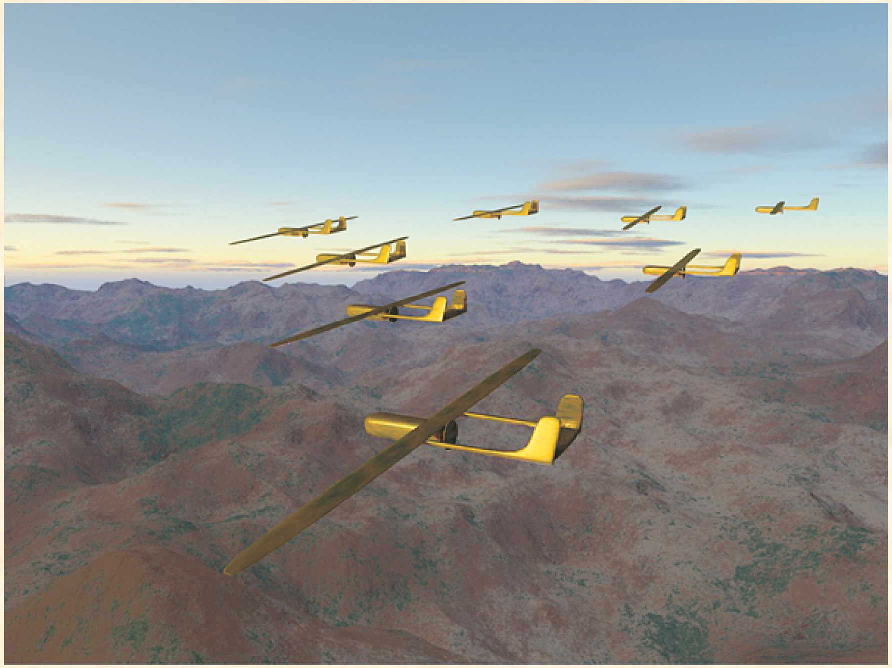
Birds Flocking

Flocking: to congregate or travel in flock

Consensus

A position reached by a group of mobile agents





Battle field management scenario

What are in common ?

- Swarming
- Flocking
- Rendezvous
- Consensus
- Synchrony
- Cooperation
- ......

⇨

Distributed coordination of a network of agents:

- ✓ Agents
- ✓ Network
- ✓ Distributed local control
- ✓ Global consensus

Synchronization in Complex Networks

Local Synchronization in Complex Networks
Master Stability Function

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \sigma \mathbf{G} \otimes \mathbf{H}(\mathbf{x})$$

- State variable: $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^N)$
- Self-nonlinear function

$$\mathbf{F}(\mathbf{x}) = [\mathbf{F}(\mathbf{x}^1), \mathbf{F}(\mathbf{x}^2), \ldots, \mathbf{F}(\mathbf{x}^N)]$$

- Nonlinear inner coupling function

$$\mathbf{H}(\mathbf{x}) = [\mathbf{H}(\mathbf{x}^1), \mathbf{H}(\mathbf{x}^2), \ldots, \mathbf{H}(\mathbf{x}^N)]$$

- Coupling matrix $\mathbf{G}$
- Coupling strength σ

L. M. Pecora T. L. Carroll, “Master stability functions for synchronized coupled systems,” Phys. Rev. Lett., vol. 80, no. 10, pp. 2109–2112, 1998.

$$\mathbf{E} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{G}_1 = \begin{pmatrix} -2 & 1 & 0 & \cdots & 1 \\ 1 & -2 & 1 & \cdots & 0 \\ 0 & 1 & -2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \cdots & 1 & -2 \end{pmatrix},$$

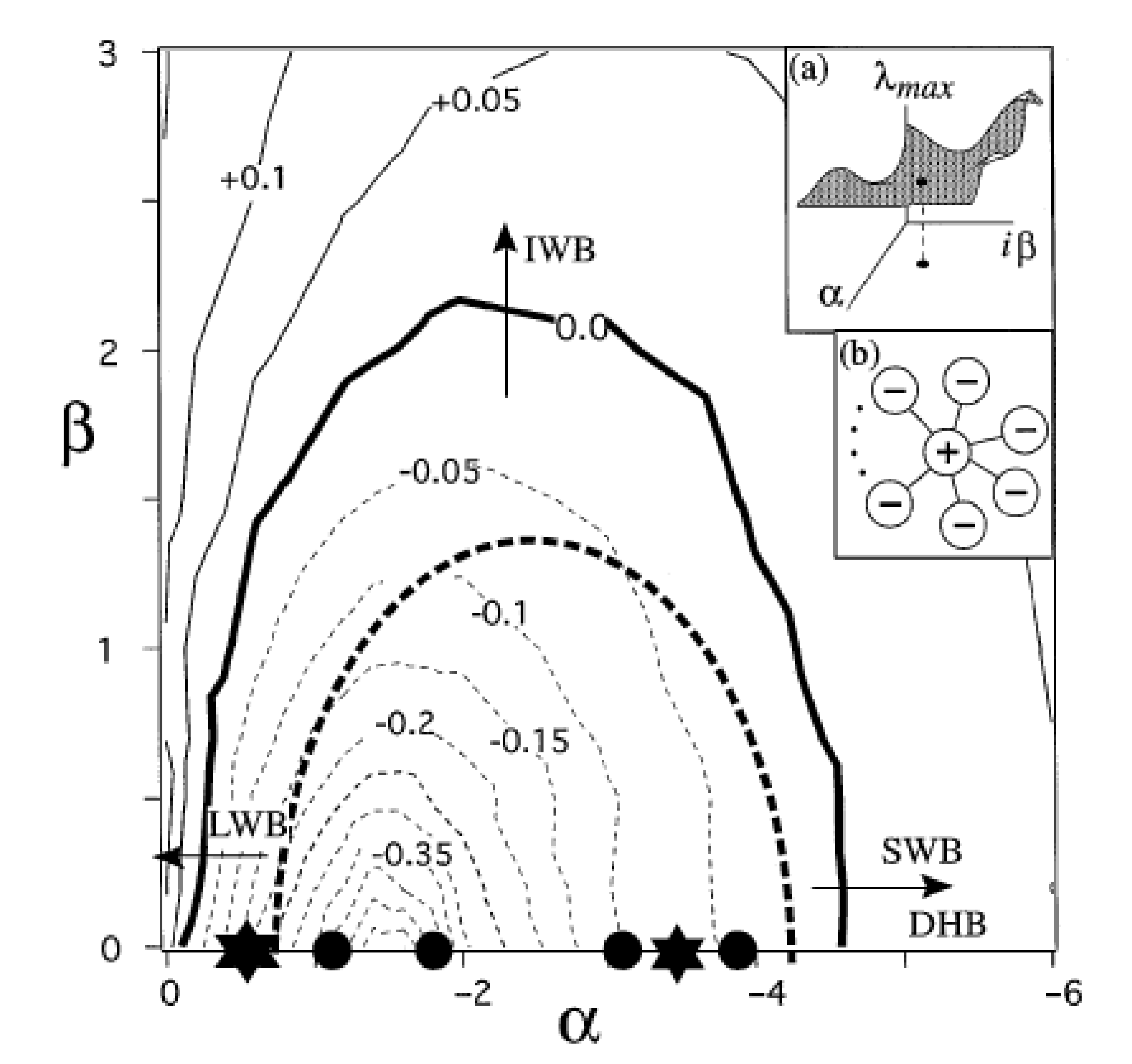
$$\mathbf{G}_2 = \begin{pmatrix} -N+1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \cdots & 0 & -1 \end{pmatrix}$$

$$\dot{\xi}_k = [D\mathbf{F} + \sigma\gamma_k D\mathbf{H}]\xi_k$$

- separating that from the other transverse directions (stability of N-1 subsystems: difficult to check)

Synchronization region

$$\dot{\zeta} = [D\mathbf{F} + (\alpha + i\beta)D\mathbf{H}]\zeta$$



Local Synchronization in Complex Networks

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + c\sum_{\substack{j=1\\ j\neq i}}^{N} a_{ij}\Gamma(\mathbf{x}_j - \mathbf{x}_i), \quad i = 1,2,\ldots,N$$

- State vector $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \Re^n$
- Self-nonlinear function $f(\mathbf{x}_i)$
- inner coupling matrix $\Gamma = \mathrm{diag}(r_1, r_2, \ldots, r_n)$
- Coupling matrix $\mathbf{A} = (a_{ij}) \in \Re^{N\times N}$
- Coupling strength c

X. Wang and G. Chen, “Synchronization in scale-free dynamical networks: robustness and fragility,” IEEE Trans. Circuits Syst. I, vol. 49, no. 1, pp. 54–62, Jan. 2002.

Global Synchronization in Complex Networks

$$\sum_{\substack{j=1\\ j\neq i}}^{N} a_{ij} = \sum_{\substack{j=1\\ j\neq i}}^{N} a_{ji} = k_i, \quad i = 1, 2, \ldots, N.$$

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + c\sum_{j=1}^{N} a_{ij}\Gamma\mathbf{x}_j, \quad i = 1, 2, \ldots, N.$$

$$a_{ii} = -k_i, \quad i = 1, 2, \ldots, N.$$

Laplacian matrix $L = (L_{ij})_{N\times N}$

$$L_{ii} = -\sum_{j=1, j\neq i}^{N} L_{ij}\,, L_{ij} = -G_{ij},\ i \neq j.(\dot{x}_i(t) = -\sum_{j=1}^{N} L_{ij}x_j(t))$$

The Laplacian matrix L has a simple eigenvalue 0 and all the other eigenvalues have positive real parts if and only if the directed network has a spanning tree.

- **W. Yu**, J. Cao, J. Lü, Global synchronization of linearly hybrid coupled networks with time-varying delay, SIAM Journal on Applied Dynamical Systems, vol. 7, no. 1, pp. 108-133, 2008.

Consensus in Multi-agent Systems

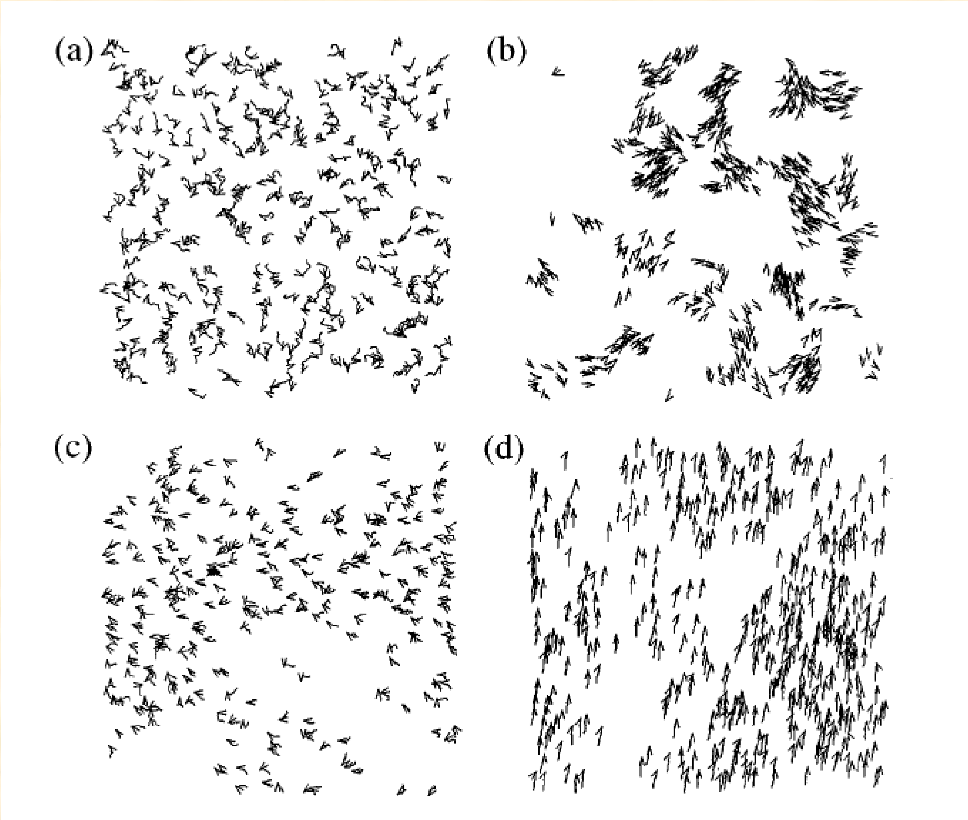
Vicsek Model

- Position:

$$x_i(t+1) = x_i(t) + v_i(t)\Delta t$$

- Heading:

$$\theta(t+1) = <\theta(t)>_r + \Delta\theta$$



(a) Initial: Random positions/velocities

(b) Low density/noise: grouped

(c) High density/noise: correlated

(d) High density / Low noise

→ coordinated motion

Vicsek, Czirok, Jacob, Cohen, Shochet, "Novel type of phase transition in a system of self-driven particles," Phys. Rev. Lett., 1995, 75 (6): 1226

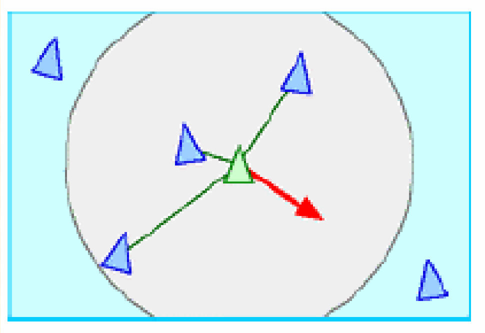
Boids Flocking Model

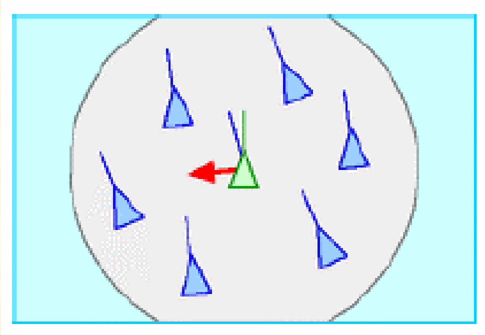
Three Rules:

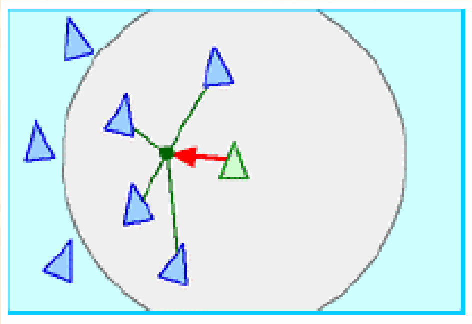
Separation: Steer to avoid crowding local flockmates

Alignment: Steer to move toward the average heading of local flockmates

Cohesion: Steer to move toward the average position of local flockmates





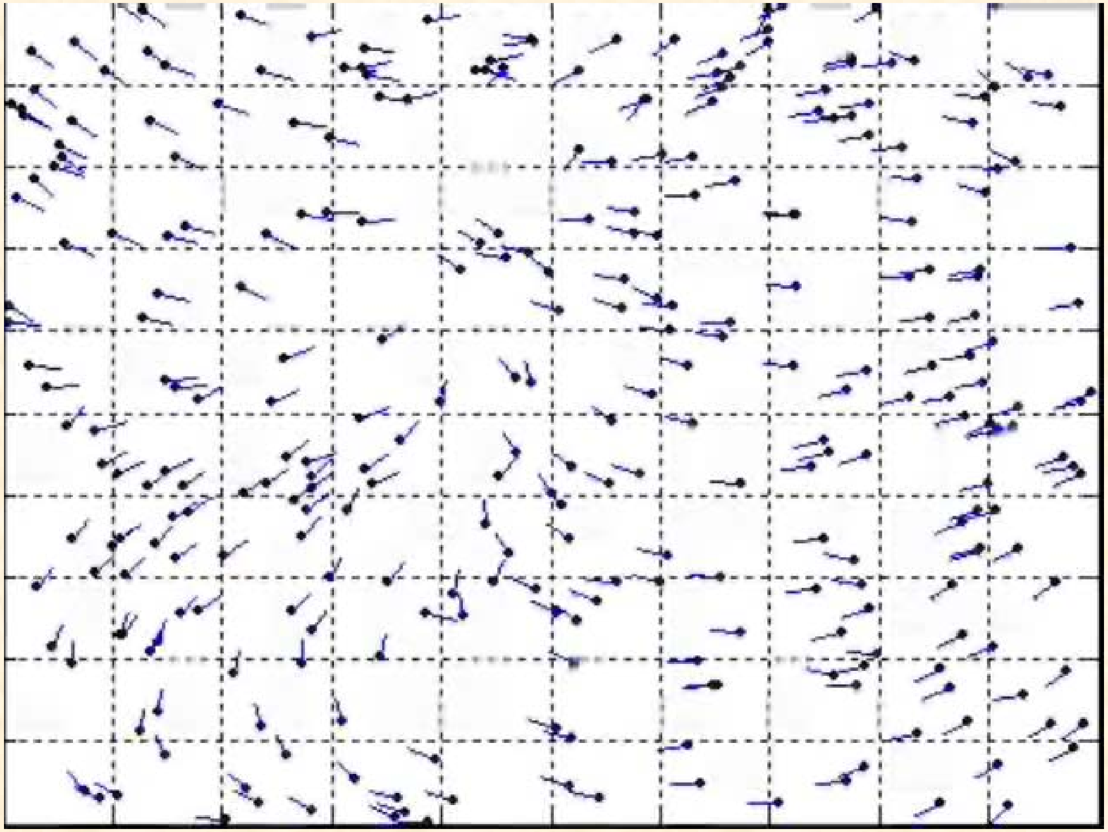


Reynolds, "Flocks, herd, and schools: A distributed behavioral model," Computer Graphics, 1987, 21(4): 15-24 http://www.red3d.com/cwr/boids/

Convergence

$$\theta(t+1) = F_{G_p(t)}\theta(t)$$

$$\lim_{t\to\infty}\theta(t) = \theta_{ss}1$$



Jadbabaie, Lin, Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," IEEE Trans. Auto. Control, 2003, 48(6): 988-1001

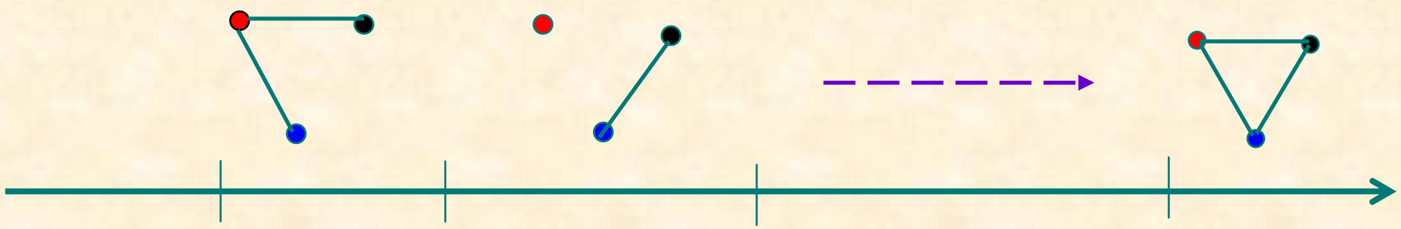
Moreau, "Stability of multiagent systems with time-dependent communication links," IEEE Trans. Auto. Control, 2005, 50(2): 169-182

Consensus Protocol

$$
\begin{aligned}
\dot{x}_i(t) &= u_i(t),\ i = 1, 2, \ldots, N. \\
u_i(t) &= \sum_{j=1, j\neq i}^{N} a_{ij}(x_j(t) - x_i(t)) = -\sum_{j=1}^{N} L_{ij} x_j(t)
\end{aligned}
$$

Design a network connection topology, or design local control law, so that $\left\| x_i - x_j \right\| \to 0$ (consensus = synchronization)

Consensus is reached asymptotically **if** there exists an infinite sequence of bounded intervals such that the union of the graphs over such intervals is totally connected.



Olfati-Saber, Murray, "Consensus problems in networks of agents with switching topology and time-delays," IEEE Trans. Auto. Control, 2004, 49(9): 1520-1533

A Unified Framework

Differences

- *Different Asymptotic States* (Nonlinear Dynamics versus Linear Dynamics)
- *Different Focuses* (Known Connectivity versus Time-varying Distributed Protocol)
- *Different Approaches* (Lyapunov Method versus Stochastic Matrix Theory)
- *Different Inner Matrices* : Γ (General Inner Matrix versus Particular Inner Matrix)

Different Asymptotic States (Nonlinear Dynamics versus Linear Dynamics).

$$\dot{x}_i(t) = f(x_i(t)) - c\sum_{j=1}^{N} L_{ij}\Gamma x_j(t)$$

Time-varying asymptotic states

Single State

$$\dot{s}(t) = f(s(t))$$

$$\begin{aligned}\dot{x}_i(t) &= u_i(t),\ i = 1, 2, \ldots, N.\\ u_i(t) &= \sum_{j=1, j\neq i}^{N} a_{ij}(x_j(t) - x_i(t)) = -\sum_{j=1}^{N} L_{ij}x_j(t)\end{aligned}$$

Fixed asymptotic states

W. Yu, G. Chen, M. Cao, "Consensus in directed networks of agents with nonlinear dynamics," IEEE Trans. Auto. Control. 2011, 56(6): 1436-1441

Different Focuses (Known Connectivity versus Time-varying Distributed Protocol)

$$\dot{x}_i(t) = f\left(x_i(t)\right) - c\sum_{j=1}^{N} L_{ij}\Gamma x_j(t)$$

$$\dot{x}_i(t) = \sum_{j\neq i} a_{ij}\left(x_j(t) - x_i(t)\right) = -\sum_{j=1}^{N} L_{ij}x_j(t).$$

Known connectivity

Designed distributed protocol

- Synchronization: connected
- Consensus: time-varying (designed)

Different Approaches (Lyapunov Method versus Stochastic Matrix Theory)

- Common approach: algebraic graph theory
- Synchronization in complex networks: Lyapunov method, matrix theory
- Consensus in multi-agent systems:

Stochastic matrix theory, convexity analysis, matrix theory

Different Inner Matrices (General Inner Matrix versus Particular Inner Matrix)

$$\dot{x}_i(t) = f(x_i(t)) - c\sum_{j=1}^{N} L_{ij}\Gamma x_j(t)$$

$$\dot{x}_i(t) = \sum_{j\neq i} a_{ij}(x_j(t) - x_i(t)) = -\sum_{j=1}^{N} L_{ij}x_j(t).$$

- First-order Γ : identity matrix
- Second-order Γ ?

$$\dot{x}_i(t) = v_i,$$

$$\dot{v}_i(t) = \alpha\sum_{j=1,j\neq i}^{N} G_{ij}(x_j(t) - x_i(t)) + \beta\sum_{j=1,j\neq i}^{N} G_{ij}(v_j(t) - v_i(t)),$$

W. Yu, G. Chen, M. Cao, "Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems," Automatica 2010, 46(6): 1089-1095

$$\dot{\eta}_i(t) = A\eta_i(t) - \sum_{j=1}^{N} L_{ij}\Gamma\eta_j(t),\ i = 1, 2, \ldots, N$$

$$\eta_i = (x_i, v_i)^T$$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} 0 & 0 \\ \alpha & \beta \end{pmatrix}$$

Second-order Consensus with Nonlinear Dynamics

$$\dot{x}_i(t) = v_i(t)$$

$$\dot{v}_i(t) = \boxed{f\left(x_i(t), v_i(t), t\right)} - \alpha \sum_{j=1}^{N} L_{ij} x_j(t) - \beta \sum_{j=1}^{N} L_{ij} v_j(t), \qquad i = 1, 2, \ldots, N$$

Nonlinear vector function

$$\eta_i = (x_i, v_i)^T, \; C = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \; D = \begin{pmatrix} 0 & 0 \\ \alpha & \beta \end{pmatrix}$$

$$\dot{\eta}_i(t) = f(x_i) + C\eta_i(t) - \sum_{j=1}^{N} L_{ij} D \eta_j(t)$$

Stabilize the system (not coupling)

Synchronization in complex networks
Wang and Chen (TCAS I 2002)

W. Yu, G. Chen, M. Cao, J. Kurths, "Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics," IEEE Trans. Sys. Man Cybern., B, 2010, 40(3): 881-891

Higher-order Consensus in linear system

$$\begin{aligned}\dot{\xi}_i^{(1)}(t) &= \xi_i^{(2)}(t),\\ &\vdots\\ \dot{\xi}_i^{(m-1)}(t) &= \xi_i^{(m)}\\ \dot{\xi}_i^{(m)}(t) &= u_i, i = 1, 2, \ldots, N,\end{aligned}$$

Designed control input

$$u_i(t) = c \sum_{j=1, j\neq i}^{N} G_{ij} \sum_{k=1}^{m} \alpha_k \left(\xi_j^{(k)}(t) - \xi_i^{(k)}(t)\right)$$

Main result (eigenvalue analysis for linear system): mth-order consensus can be achieved if and only if

$$c\lambda_i \in \mathcal{S}, i = 2, \ldots, N,$$

where $\mathcal{S} = \bigcup_{i \in \mathcal{N}} \mathcal{S}_i$ and $\mathcal{N} = \{i | \mathcal{S}_i$ is a stable consensus region, $i = 1, \ldots, r + 1\}$.

The stable consensus region depends on network parameters

W. Yu, G. Chen, W. Ren, J. Kurths, W. Zheng, "Distributed Higher Order Consensus Protocols in Multiagent Dynamical Systems," IEEE Trans. Cir. Syst. I 2011, in press

$$\dot{\eta}_i(t) = g(\eta_i) - c\sum_{j=1}^{N} L_{ij} D\eta_j(t), i = 1, 2, \ldots, N,$$

$$\eta_i = (\xi_i^{(1)}, \ldots, \xi_i^{(m)})^T$$

$$C = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{m\times m}$$

Γ

$$D = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_m \end{pmatrix}_{m\times m}$$

Consensus in Multi-agent Systems

General Consensus Protocols with Linear and Nonlinear Dynamics

- [1] **W. Yu**, G. Chen, M. Cao, Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems, ***Automatica***, vol. 46, no. 6, pp. 1089-1095, 2010.（**ESI**高被引论文）
- [2] **W. Yu**, G. Chen, W. Ren, J. Kurths, W. Zheng, Distributed higher-order consensus protocols in multi-agent dynamical systems, ***IEEE Trans. Circuits and Systems I***, vol. 58, no. 8, pp. 1924-1932, 2011. **(**2009年全复杂网络学术会议最佳学生论文奖**)**
- [3] **W. Yu**, G. Chen, M. Cao, Consensus in directed networks of agents with nonlinear dynamics, ***IEEE Trans. Automatic Control***, vol. 56, no. 6, pp. 1436-1441, 2011.
- [4] **W. Yu**, G. Chen, M. Cao, J. Kurths, Second-order consensus for multi-agent systems with directed topologies and nonlinear dynamics, ***IEEE Trans. Systems, Man, and Cybernetics-Part B***, vol. 40, no. 3, pp. 881-891, 2010.（**ESI**高被引论文）

Flocking

- [5] **W. Yu**, G. Chen, M. Cao, Distributed leader-follower flocking control for multi-agent dynamical systems with time-varying velocities, ***Systems & Control Letters***, vol. 59, no. 9, pp. 543-552, 2010.
- [6] J. Zhou, **W. Yu**, X. Wu, M. Small, J. Lu, Flocking of multi-agent dynamical systems on pseudo-leader mechanism, ***Systems & Control Letters***, vol. 61, no. 1, pp. 195-202, 2012.

Pinning, Intermittent, and Sampling Control

- [7] Q. Song, J. Cao, **W. Yu**, Second-order leader-following consensus of nonlinear multi-agents via pinning control, ***Systems & Control Letters***, vol. 59, no. 9, pp. 553-562, 2010.
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Filtering

- [9] **W. Yu**, G. Chen, Z. Wang, W. Yang, Distributed consensus filters in sensor networks, ***IEEE Trans. Systems, Man, and Cybernetics-Part B***, vol. 39, no. 6, pp. 1568-1577, 2009. （**ESI**高被引论文）

Synchronization in Complex Networks

General Synchronization Criteria

- [1] W. Xiong, **W. Yu**, Synchronization of continuous complex networks based on asynchronously discontinuous controllers, ***Chaos***, vol. 21, no. 2, 023120, 2011.
- [2] **W. Yu**, J. Cao, G. Chen, J. Lü, J. Han, W. Wei, Local synchronization of a complex network model, ***IEEE Trans. Systems, Man, and Cybernetics-Part B***, vol. 39, no. 1, pp. 230-241, 2009.（**ESI**高被引论文）
- [3] T. Huang, C. Li, **W. Yu**, G. Chen, Synchronization of delayed chaotic systems with parameter mismatches by using intermittent linear state feedback, ***Nonlinearity***, vol. 22, pp. 569-584, 2009.（**ESI**高被引论文）
- [4] **W. Yu**, J. Cao, J. Lü, Global synchronization of linearly hybrid coupled networks with time-varying delay, ***SIAM Journal on Applied Dynamical Systems***, vol. 7, no. 1, pp. 108-133, 2008.（**ESI**高被引论文；**2008**年第三届中国百篇最具影响国际学术论文）

Pinning Synchronization

- [5] Q. Song, F. Liu, J. Cao, **W. Yu**, Pinning-controllability analysis of complex networks: an m-matrix approach, ***IEEE Trans. Circuits and Systems I***, in press.
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Adaptive Synchronization

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- [9] J. Zhu, M. Zhao, **W. Yu**, C. Zhou, B. Wang, Better synchronizability in generalized adaptive networks, ***Physical Review E***, vol. 81, 026201, 2010.
- [10] J. Zhou, **W. Yu**, X. Li, M. Small, J. Lu, Identifying the topology of a coupled FitzHugh-Nagumo neurobiological network via a pinning mechanism***IEEE Trans. Neural Networks***, vol. 20, no. 10, pp. 1679-1684, 2009.

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