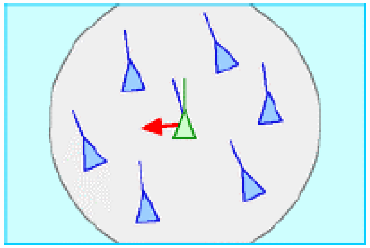
Outline

Flocking without connectivity preserving

Flocking with connectivity preserving

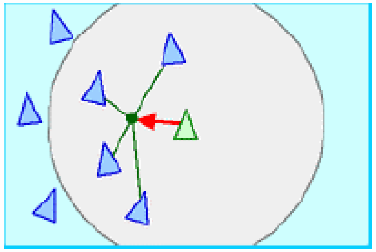
Classical Boids Model

Velocity Matching (Alignment)



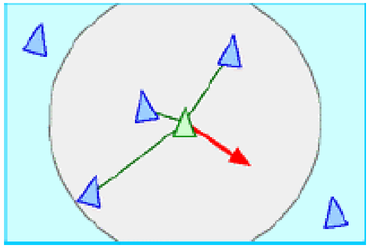
attempt to match velocity with nearby flockmates

Flock Centering (Cohesion)



Stay close to nearby flockmates

Collision Avoidance (Separation)



avoid collisions with nearby agents

Reynolds , "Flocks, Herd, and Schools: A Distributed Behavioral Model", Computer Graphics, 21(4),1987.

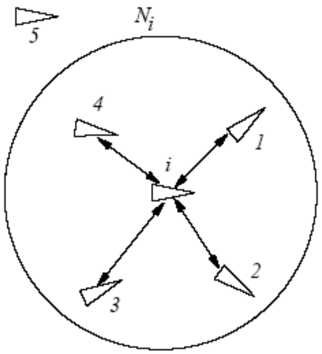
Problem Description

$$\dot{q}_i = p_i$$

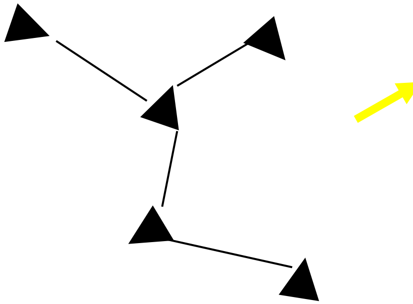
$$\dot{p}_i = u_i, \quad i = 1, \ldots, N$$

q_i **Position**

p_i **Velocity**



Goals of Control:



Velocity Alignment $\| p_i - p_j \| = 0$

Cohesion $\| q_i - q_j \| \approx d > 0$

Separation $\forall j \in N_i$

Tracking $\| p_i - p_\gamma \| = 0$

Leader:

$$\begin{cases} \dot{q}_\gamma = p_\gamma \\ \dot{p}_\gamma = f_\gamma(q_\gamma, p_\gamma) \end{cases}$$

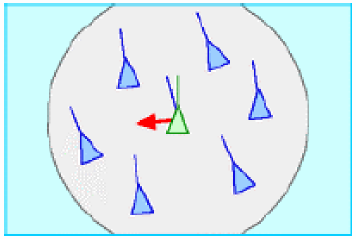
Alignment

$$\dot{q}_i = p_i$$

$$\dot{p}_i = u_i, \ i = 1, \dots, N$$

q_i **Position**

p_i **Velocity**



Goal of Control: **Alignment** $\| p_i - p_j \| = 0$

$$\dot{p}_i = u_i = -\sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$$

Synchronization Model $\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j$

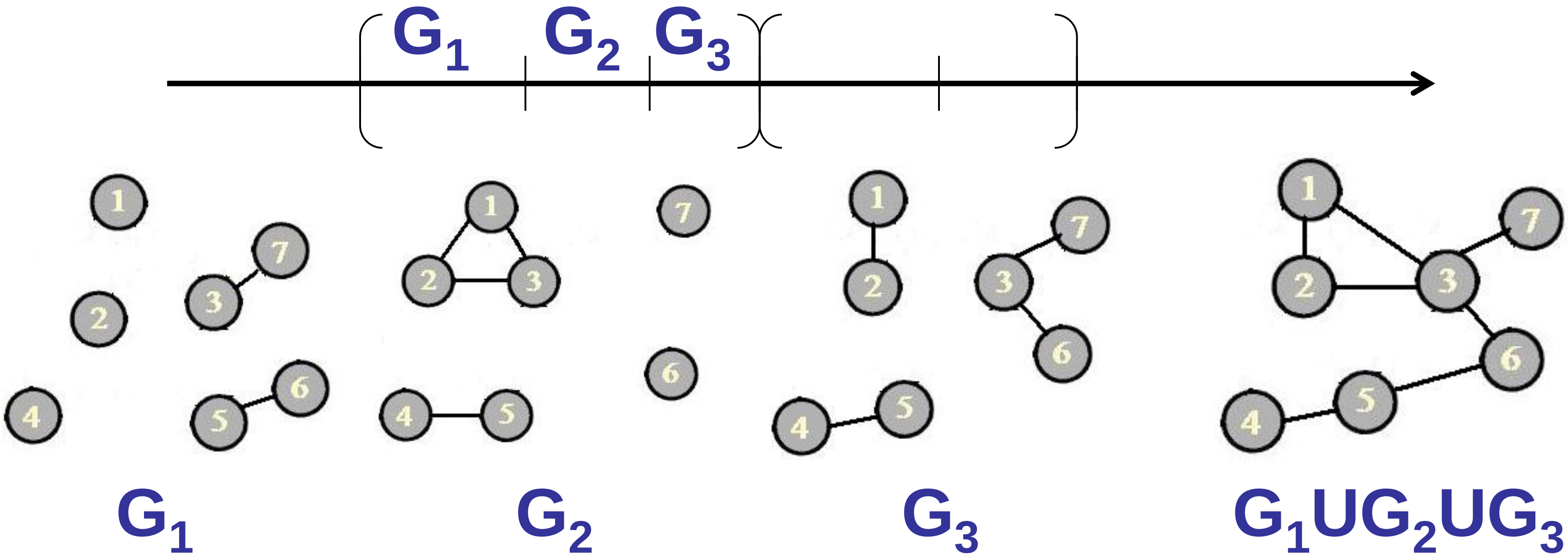
$$\dot{p}_i = \sum_{j=1}^{N} a_{ij}(t) p_j$$

Consensus Condition

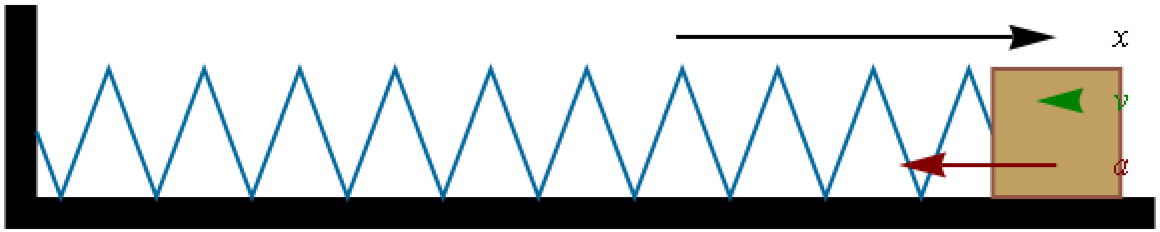
$$\dot{p}_i = u_i = -\sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$$

Strong condition: G(t) is connected for any t

Weak condition: There exists infinitely many consecutive uniformly bounded time intervals such that the union of the graph across each interval is connected



An Example: Harmonic oscillator



$$\dot{q}_i = p_i,$$
$$\dot{p}_i = -\omega^2 q_i + u_i, \quad i = 1, 2, \cdots, N,$$

$$u_i = -\sum_{i=1}^{N} a_{ij}(t)\,(p_i - p_j)$$

H Su, X Wang, Z Lin, Automatica，2009

Consensus with Input Saturation

$$\dot{x}_i = Ax_i + B\sigma(u_i), \quad i = 1, 2, \ldots, N,$$

$$\sigma(u_i) = [\mathrm{sat}(u_{i1}) \ \ \mathrm{sat}(u_{i2}) \ \ \cdots \ \ \mathrm{sat}(u_{im})]^{\mathrm{T}}, \quad \mathrm{sat}(u_{ij}) = \mathrm{sgn}(u_{ij}) \min\{|u_{ij}|, \varpi\},$$

$$\dot{x}_{N+1} = Ax_{N+1}.$$

$$\lim_{t \to \infty} \|x_i(t) - x_{N+1}(t)\| = 0, \quad i = 1, 2, \ldots, N,$$

The pair (A, B) is asymptotically null controllable with bounded controls, that is,

(1) (A, B) is stabilizable;

(2) All the eigenvalues of A are in the closed left-half s-plane.

Consensus with Input Saturation

$$\dot{x}_i = Ax_i + B\sigma(u_i), \quad i = 1, 2, \ldots, N,$$

$$A^{\mathrm{T}}P + PA - PBB^{\mathrm{T}}P + \varepsilon I = 0 \qquad \lim_{\varepsilon \to 0} P(\varepsilon) = 0$$

Step 1. Solve the parametric algebraic Riccati equation (ARE)

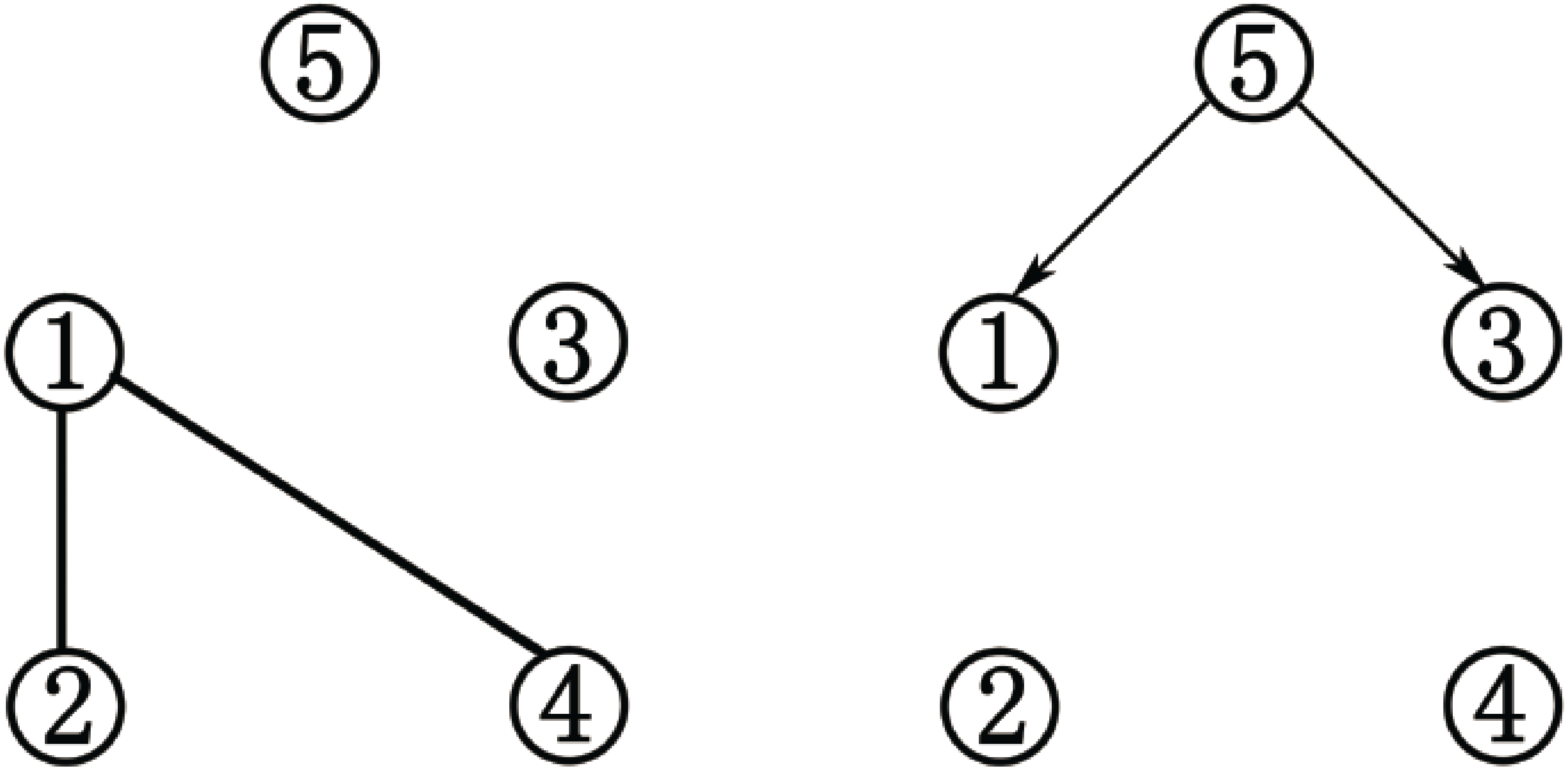
$$A^{\mathrm{T}}P + PA - 2\gamma PBB^{\mathrm{T}}P + \varepsilon I = 0, \quad \varepsilon \in (0, 1],$$

where $\gamma \leq \min\{\lambda_1(L_s + H)\}$ is a positive constant.

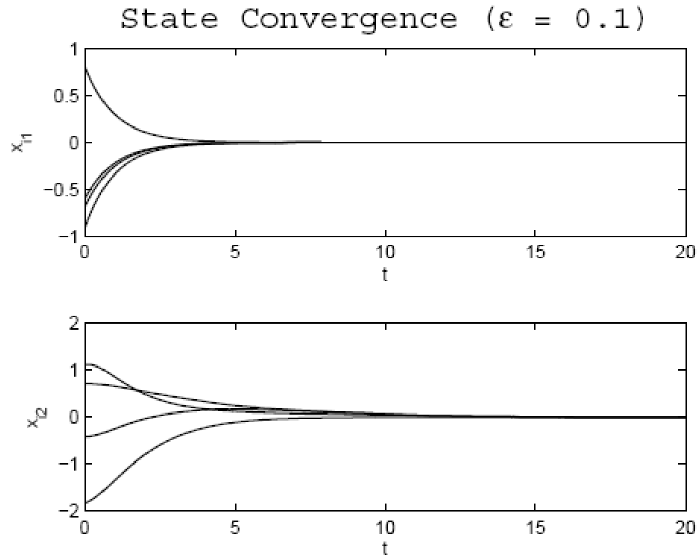
Step 2. Construct a linear feedback law for agent i as

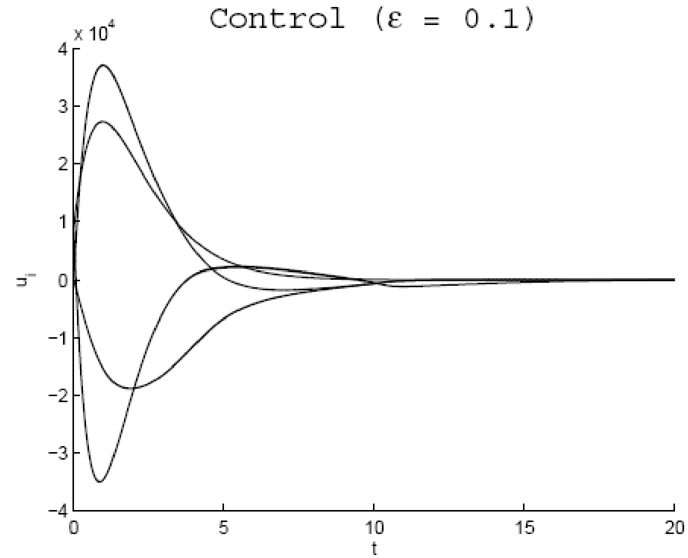
$$u_i = -B^{\mathrm{T}}P(\varepsilon)\sum_{j=1}^{N} a_{ij}(t)(x_i - x_j) - B^{\mathrm{T}}P(\varepsilon)h_i(t)(x_i - x_{N+1}), \quad i = 1, 2, \ldots, N.$$

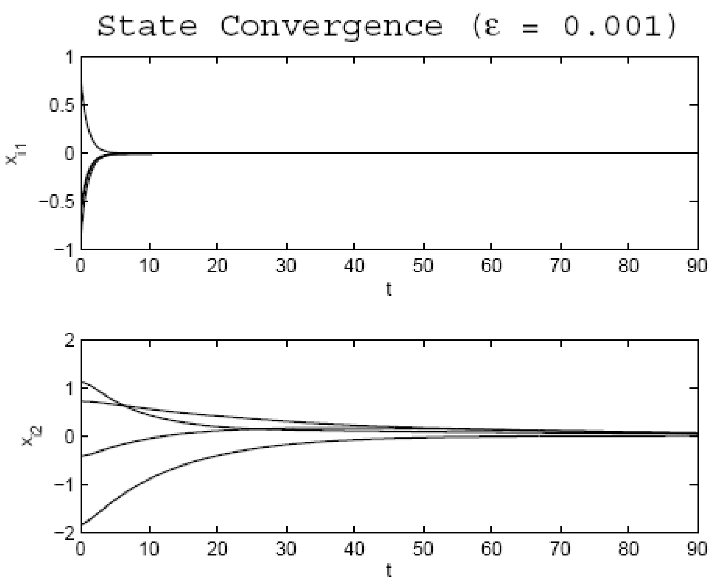
Consensus with Input Saturation

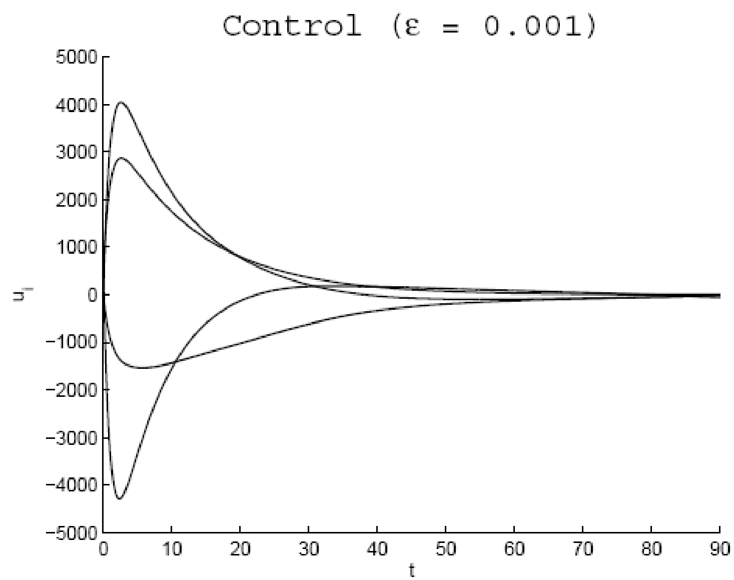


Consensus with Input Saturation



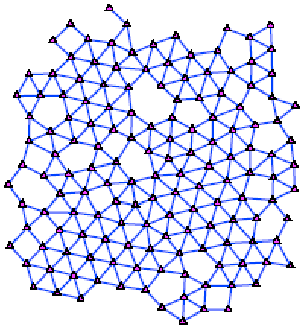






Separation & Cohesion

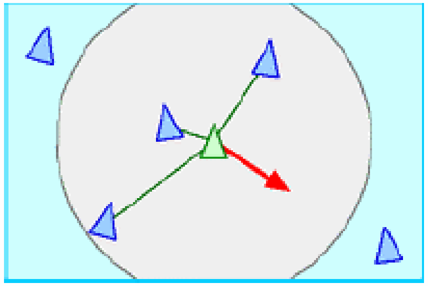
Artificial Potential Function Method

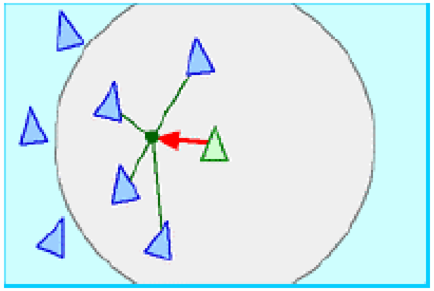


Goal of Control:

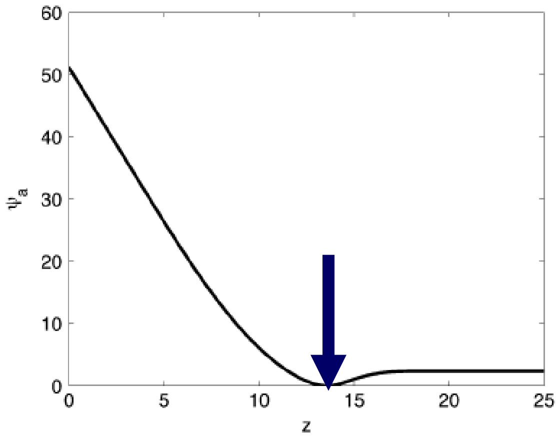
$$\| q_i - q_j \| \approx d, \ \ \forall j \in N_i$$

a combination of attraction and repulsion





$$u_i = -\sum_{j \in N_i} \nabla_{q_i} \psi_\alpha (q_{ij})$$



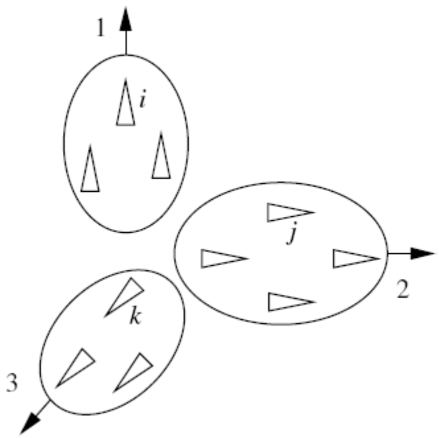
Olfati-Saber, IEEE Trans AC，2006

Previous Algorithm

$$\dot{q}_i = p_i \qquad \dot{p}_i = u_i$$

$$u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|) - \sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$$

$$-c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma)$$

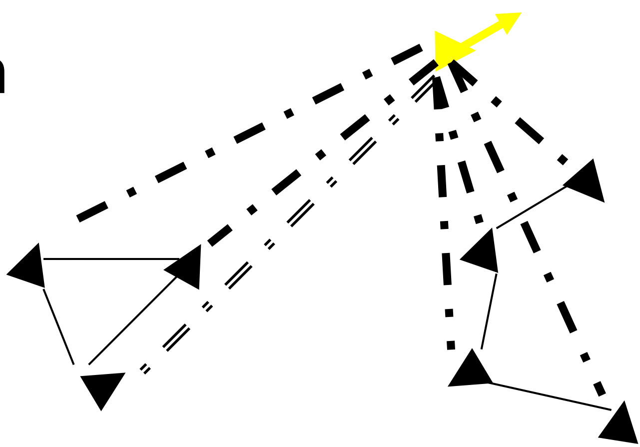


Without a leader:

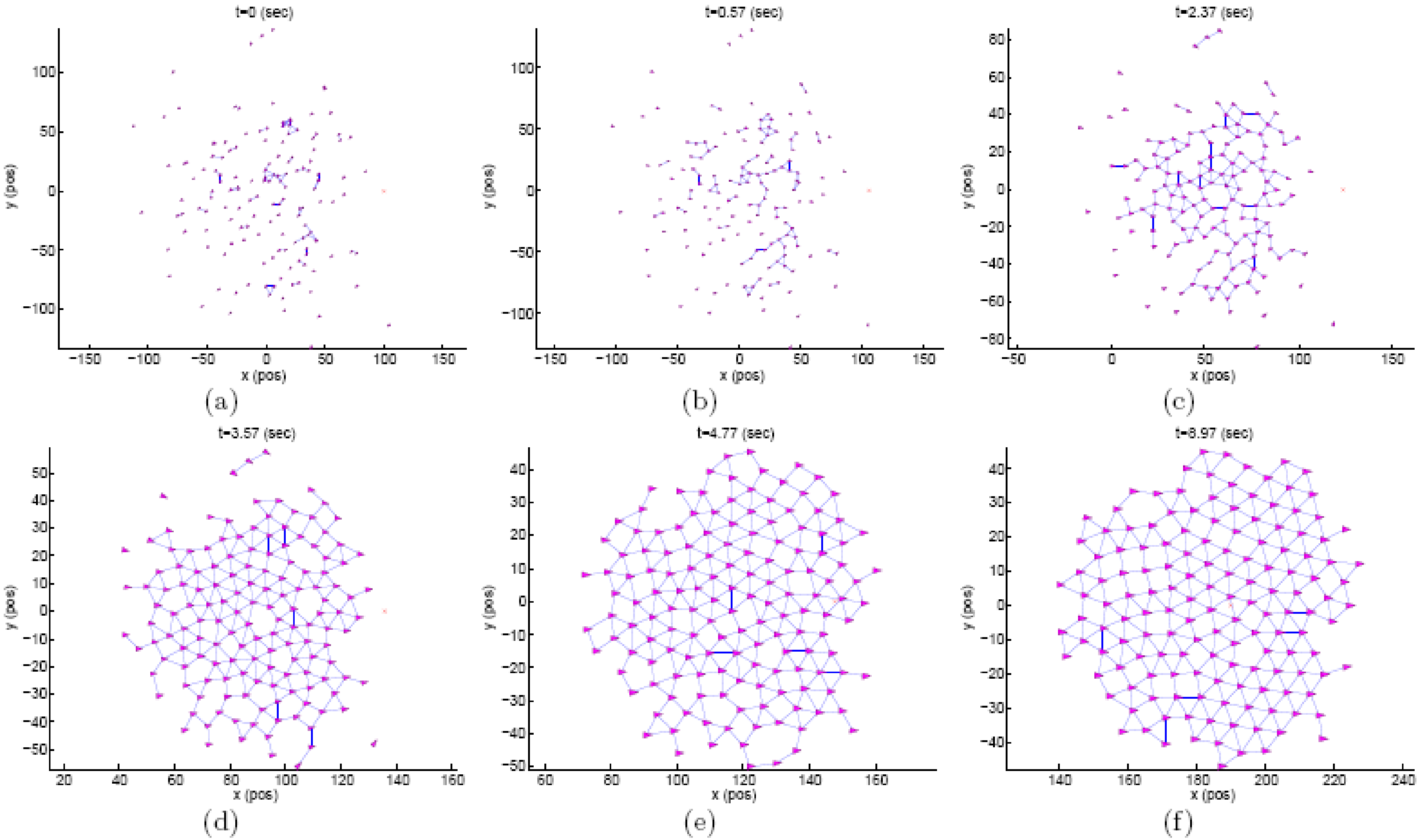
Initial connected → **Fragmentation**

With a leader:

Initial disconnected → **Flocking**



Simulations

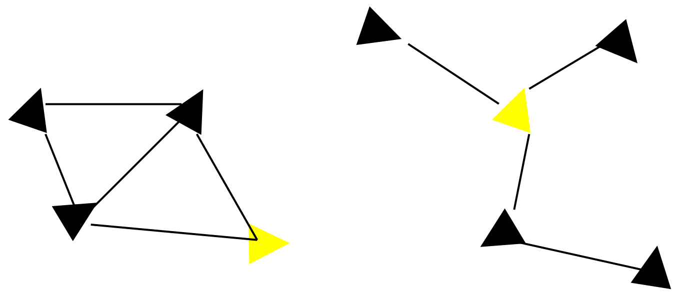


Initial positions are chosen randomly so that the initial net is highly disconnected.
No. of edges increases and has a tendency to render the net connected.

Flocking with minority of informed agents (pinning control)

Only about 5% of the bees within a honeybee swarm can guide the group to a new nest site

Flocking with minority of informed agents



(q_γ, p_γ)

Virtual leader:

$$\begin{cases} \dot{q}_\gamma = p_\gamma \\ \dot{p}_\gamma = 0 \end{cases}$$

Uninformed agent

$$u_i = \sum_{j \in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma) \frac{q_j - q_i}{\sqrt{1 - \varepsilon \|q_j - q_i\|^2}} + \sum_{j \in N_i} (p_j - p_i)$$

Separation & Cohesion **Alignment**

Informed agent

$$u_i = \sum_{j \in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma) \frac{q_j - q_i}{\sqrt{1 - \varepsilon \|q_j - q_i\|^2}} + \sum_{j \in N_i} (p_j - p_i)$$

$$+ c_1(q_\gamma - q_i) + c_2(p_\gamma - p_i)$$

Tracking

H Su, X Wang, Z Lin, Flocking of Multi-Agents with a virtual leader, IEEE T-AC，2009

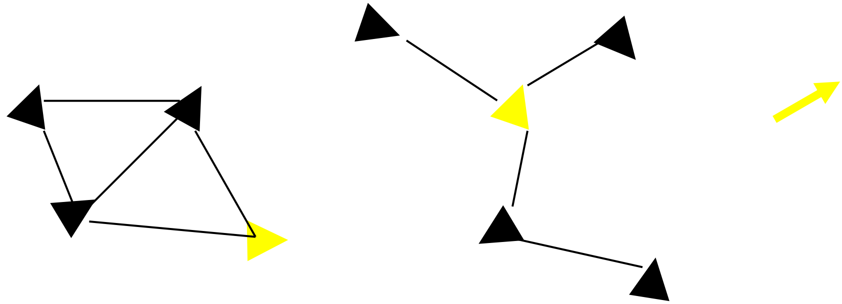
Cohesion and Velocity Matching of Informed Agents

$$u_i = \sum_{j\in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma)\frac{q_j - q_i}{\sqrt{1-\varepsilon\|q_j - q_i\|^2}} + \sum_{j\in N_i}(p_j - p_i) + c_1(q_\gamma - q_i) + c_2(p_\gamma - p_i)$$

Suppose that the initial energy Q_0 is finite.

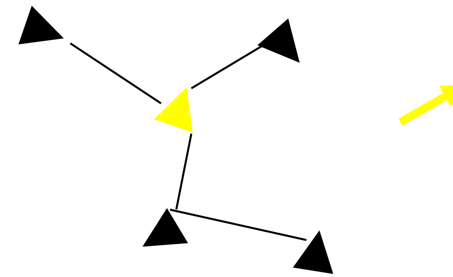
i) The distance between any informed agent and the virtual leader is not larger than $\sqrt{2Q_0/c_1}$ for all $t \geq 0$

ii) All informed agents asymptotically move with the desired velocity p_γ .



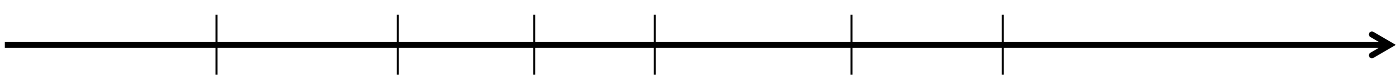
Cohesion & Velocity Matching of Uninformed Agents

$$u_i = \sum_{j\in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma) \frac{q_j - q_i}{\sqrt{1-\varepsilon\|q_j - q_i\|^2}} + \sum_{j\in N_i}(p_j - p_i)$$



Strong condition: **The uninformed agent is influenced by at least one informed agent at any time.**

Weak condition: **It gets in touch with an informed agent from time to time, directly or indirectly**



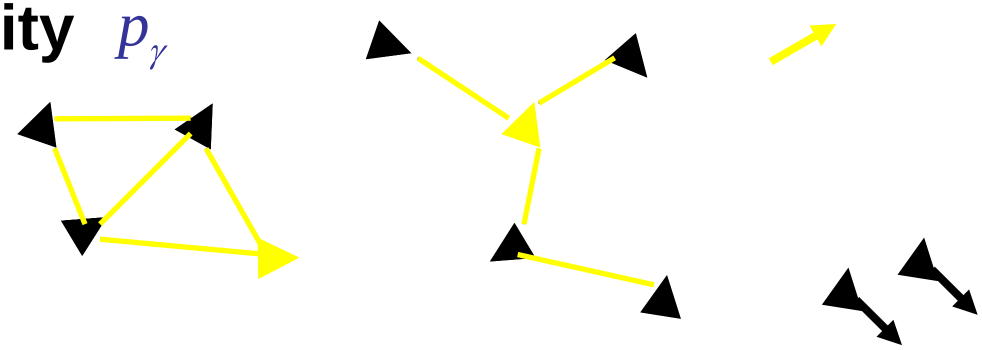
Type I uninformed agent: **If exists an infinite sequence of contiguous, nonempty and uniformly bounded time-intervals such that across each time interval there exists a joint path between this agent and one informed agent.**

Cohesive & Velocity Matching of Uninformed Agents

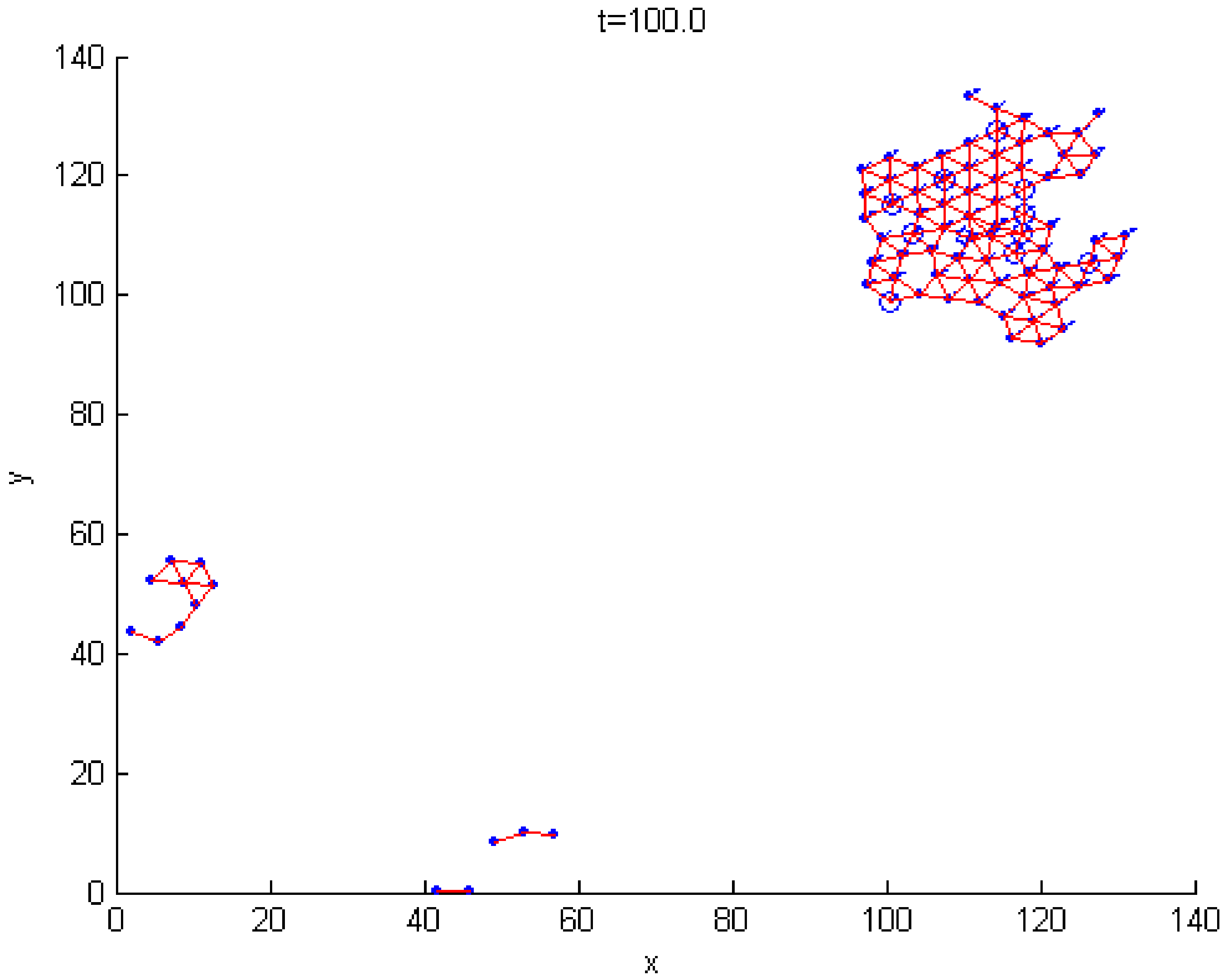
$$u_i = \sum_{j \in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma) \frac{q_j - q_i}{\sqrt{1 - \varepsilon \|q_j - q_i\|^2}} + \sum_{j \in N_i} (p_j - p_i)$$

iii) The distance between an Type-I uninformed agent and the virtual leader is bounded by a constant

iv) Each Type-I uninformed agent asymptotically moves with the desired velocity p_γ

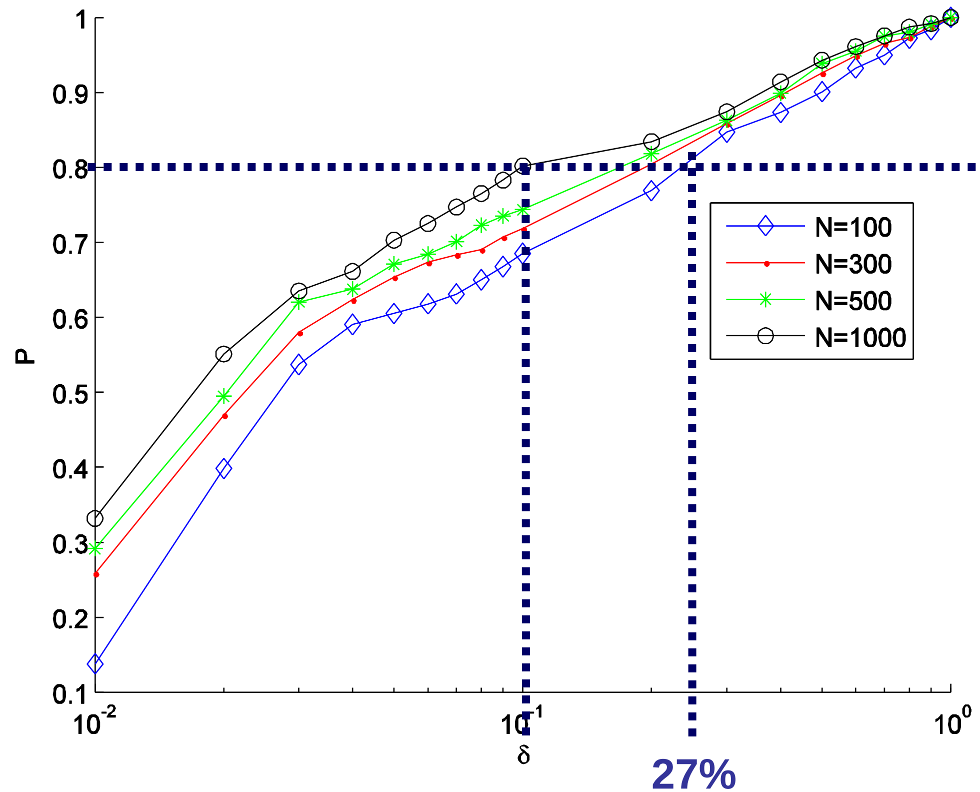


Simulation Results: N=100, M_0=10



Simulations

Fraction of agents that eventually move with the desired velocity



Fraction of randomly chosen informed agents

A very small group of informed agents can cause most of the agents to move with the desired velocity.

Simulation platform

Click here

Outline

Flocking without connectivity preserving

Flocking with connectivity preserving

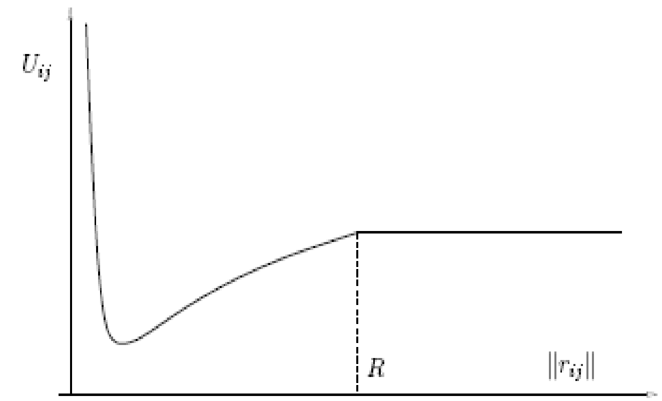
Preserving Connectivity: Basic Idea

G(0) connected ⇒ G(t) connected for all t

Our goal: Once an edge is added, it will not be lost.

Why previous algorithms fail?

$$u_i = -\sum_{j\in N_i(t)} \nabla_{q_i}\psi(\|q_{ij}\|) - \sum_{j\in N_i(t)} w_{ij}(p_i - p_j)$$



Energy: $$W = \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in N_i(t)} \psi(\|q_{ij}\|) + p_i^T p_i\right)$$

$\psi(\|q_{ij}\|) \to \infty$ **as** $\|q_{ij}\| \to 0$ $\psi(\|q_{ij}\|) = c > 0$ **as** $\|q_{ij}\| \geq r$

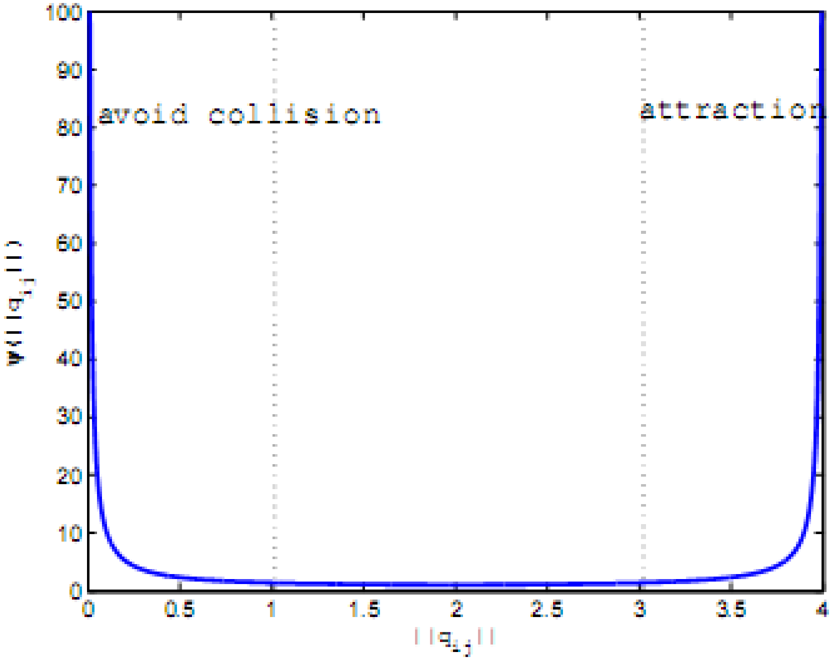
Olfati-Saber, IEEE T-AC，2006; Tanner er al., IEEE T-AC, 2007

Preserving Connectivity: Basic Idea

Our goal: Once an edge is added, it will not be lost.

A simple idea:

$$\psi(\|q_{ij}\|) \to \infty \quad \textbf{as} \quad \|q_{ij}\| \to 0 \; \textbf{or} \; \|q_{ij}\| \to r$$

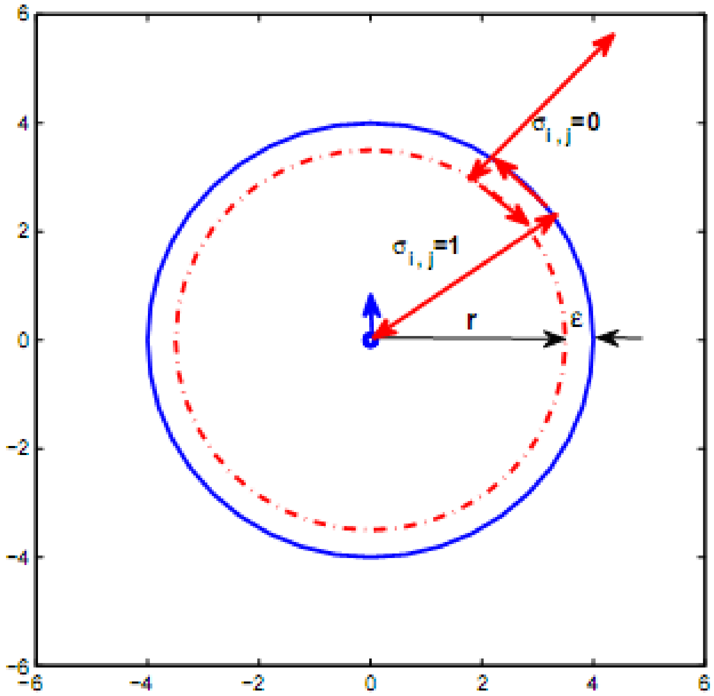


Neighboring Set

$$(i,\ j) \notin E(t) \quad \textbf{if} \quad \left\| q_i(t) - q_j(t) \right\| \geq r \qquad \varepsilon \in (0, r)$$

$$(i,\ j) \in E(t) \ \textbf{if} \ \left\| q_i(t) - q_j(t) \right\| < r - \varepsilon \quad (i,\ j) \notin E(t^-)$$

Hysteresis in addition of new links



Artificial Potential Function

$$\psi(\|q_{ij}\|) \rightarrow \infty \text{ as } \|q_{ij}\| \rightarrow 0 \text{ or } \|q_{ij}\| \rightarrow r$$

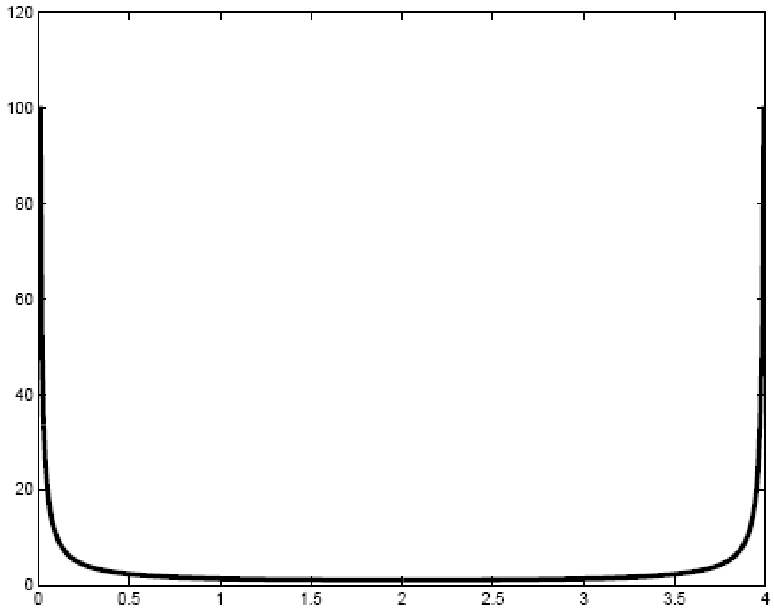
It attains unique minimum when $\|q_{ij}\|$ equals to the desired distance.

Zavlanos *et al.* 2007

$$\psi(\|q_{ij}\|) = \frac{r^2}{\|q_{ij}\|_2^2 \left(r^2 - \|q_{ij}\|_2^2\right)}$$

Our example

$$\psi(\|q_{ij}\|) = \frac{r}{\|q_{ij}\| \left(r - \|q_{ij}\|\right)}$$



Bounded Artificial Potential Function

The nonnegative potential $\psi(\|q_{ij}\|)$ is defined to be a function of the distance $\|q_{ij}\|$ between agent i and agent j, differentiable with respect to $\|q_{ij}\| \in [0, r]$ such that

(i) $\frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} > 0$ for $\|q_{ij}\| \in (0, r)$;

(ii) $\lim_{\|q_{ij}\| \to 0} \left(\frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} \cdot \frac{1}{\|q_{ij}\|} \right)$ is nonnegative and bounded;

(iii) $\psi(r) = \hat{Q} \in [Q_{\max}, +\infty)$, where $Q_{\max} \triangleq \frac{1}{2} \sum_{i=1}^{N} p_i^{T}(0) p_i(0) + \frac{N(N-1)}{2} \psi(\|r - \varepsilon_0\|)$.

$$\psi(\|q_{ij}\|) = \frac{\|q_{ij}\|^2}{r - \|q_{ij}\| + \frac{r^2}{\hat{Q}}}, \quad \|q_{ij}\| \in [0, r].$$

H Su, X Wang, G Chen, SCL，2010

Main Result

Motion equations $\dot{q}_i = p_i \quad \dot{p}_i = u_i \quad i = 1, \ldots, N$

Control law $u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|) - \sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$

where $\psi(\|q_{ij}\|) \to \infty$ **as** $\|q_{ij}\| \to 0$ **or** $\|q_{ij}\| \to r$

$$N_i(t) = \{ j \mid \sigma(i,j)[t] = 1, j \neq i, j = 1, \cdots, N \}$$

Suppose that G(0) is connected. Then

1) G(t) will be connected for all t;

2) All agents asymptotically move with same velocity;

3) Collisions among agents are avoided.

Flocking Control Using Only Position Measurements

$$u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|) - \sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$$

$$u_i = -\sum_{j \in N_i} \nabla_{q_i} \psi(\|q_j - q_i\|) - \sum_{j \in N_i} w_{ij}(y_i - y_j)$$

$$y_i = PT\hat{x}_i + P\sum_{j \in N_i} w_{ij}(q_i - q_j)$$

$$\dot{\hat{x}}_i = T\hat{x}_i + \sum_{j \in N_i} w_{ij}(q_i - q_j)$$

T: a Hurwitz matrix; P, Q: PD matrices

$$T^T P + PT = -Q$$

H Su, X Wang, G Chen, IJC，2009

Main Result

Suppose that $G(0)$ is connected. Then

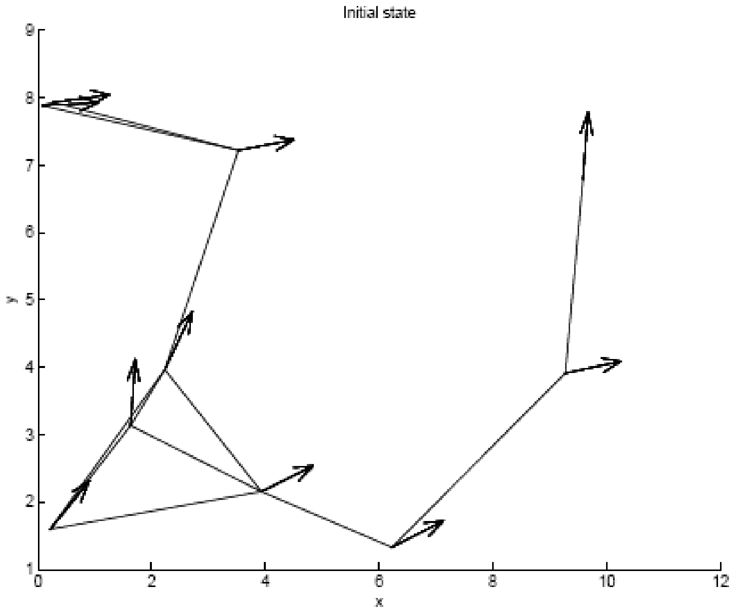
1) $G(t)$ will be connected for all t;

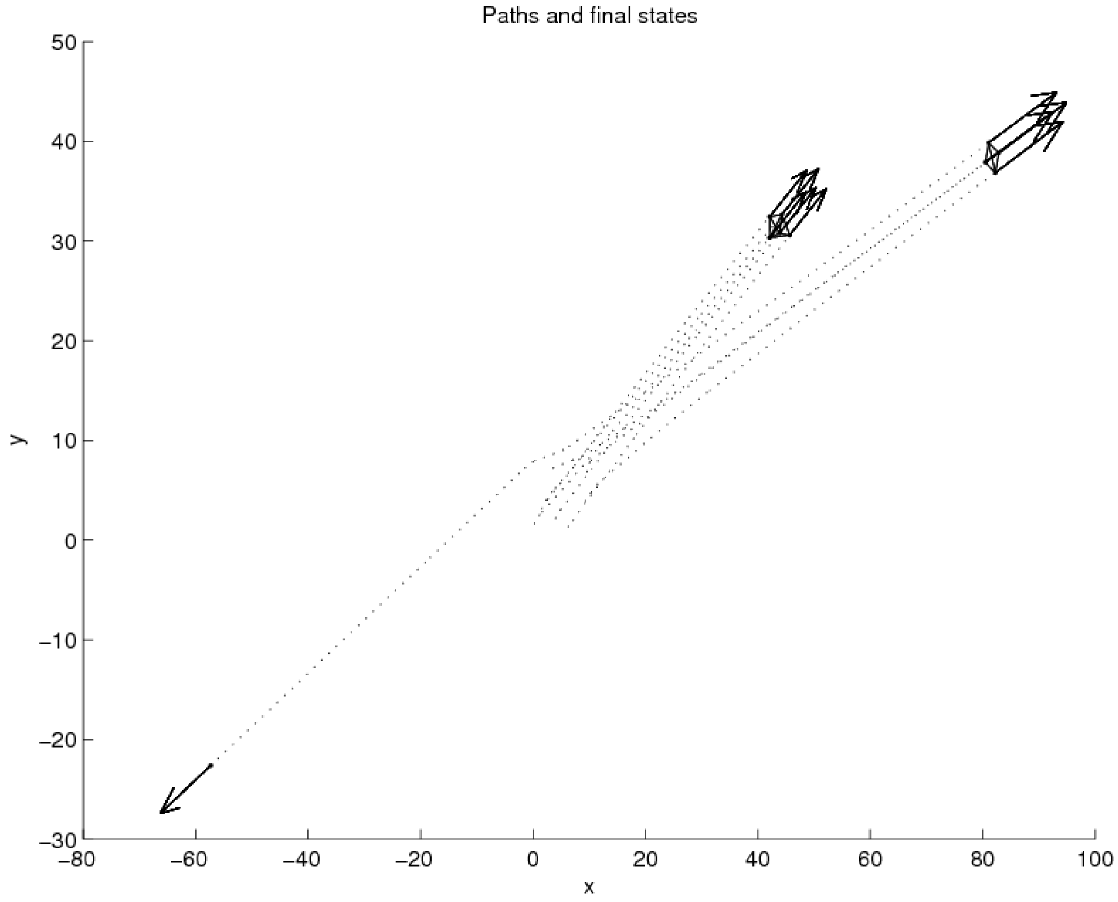
2) All agents asymptotically move with same velocity;

3) Collisions among agents are avoided.

Simulation -Previous Algorithm

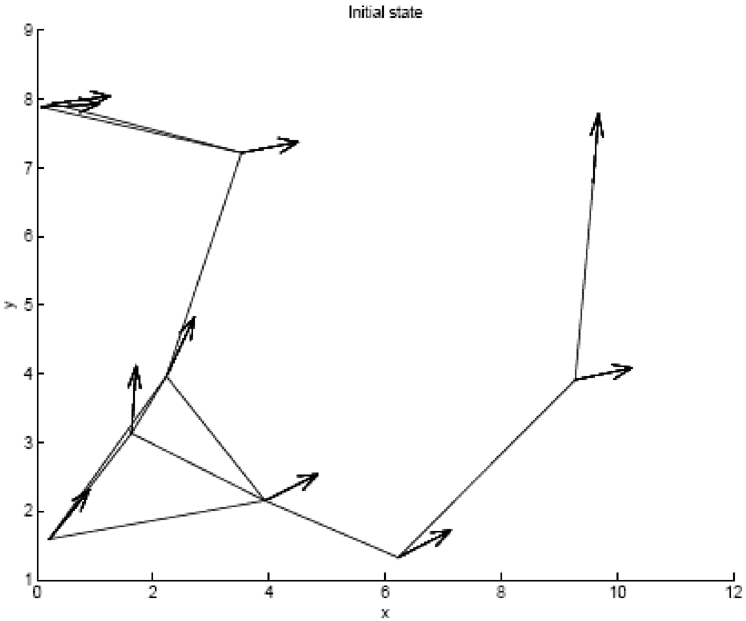
$\psi(\|q_{ij}\|) \rightarrow \infty$ **as** $\|q_{ij}\| \rightarrow 0$ $\psi(\|q_{ij}\|) = c > 0$ **as** $\|q_{ij}\| \geq r$

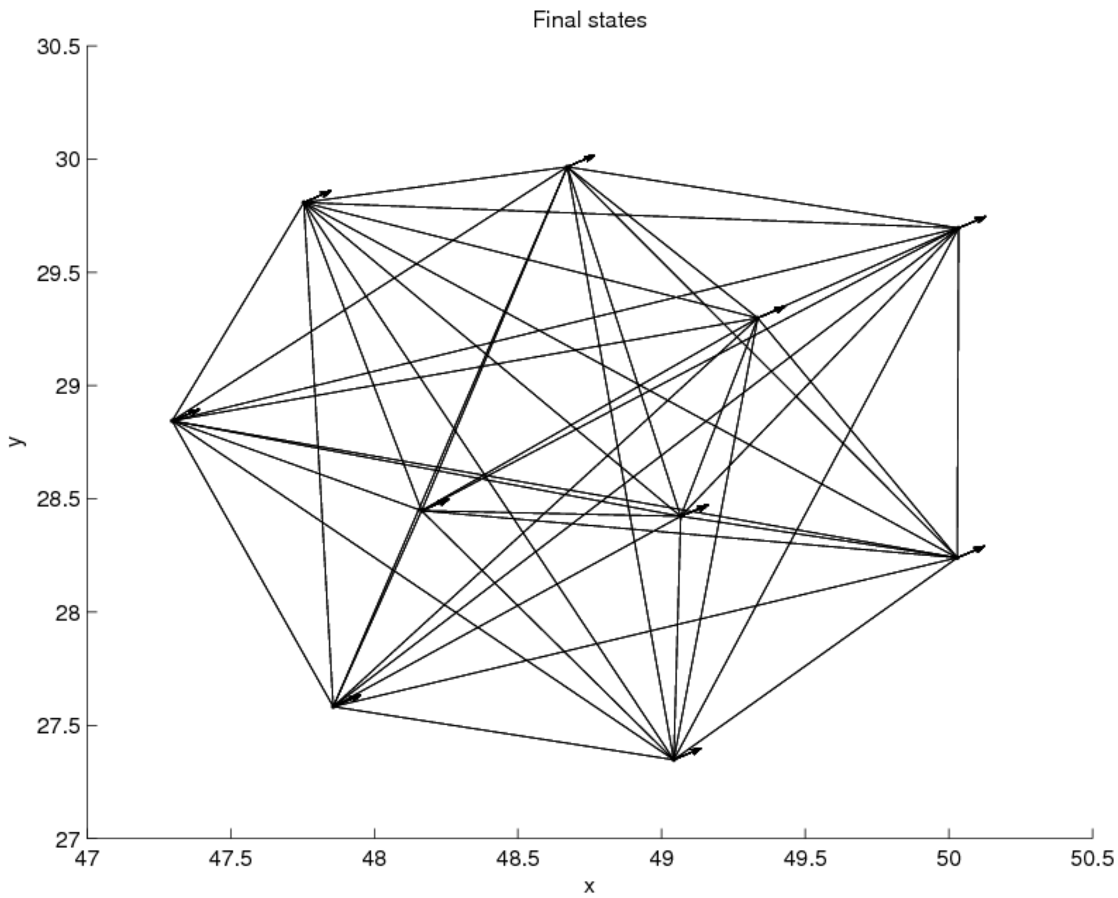




Connectivity Preserving + Observer Algorithm

$$\psi(\|q_{ij}\|) \to \infty \text{ as } \|q_{ij}\| \to 0 \text{ or } \|q_{ij}\| \to r$$





Flocking with connectivity preserving and a single informed agent

$$u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|) - \sum_{j \in N_i(t)} w_{ij}(p_i - p_j) - h_i c_1 (q_i - q_\gamma) - h_i c_2 (p_i - p_\gamma)$$

$$\psi(\|q_{ij}\|) \rightarrow \infty \ \textbf{ as } \ \|q_{ij}\| \rightarrow 0 \ \textbf{or} \ \|q_{ij}\| \rightarrow r$$

h_i=1 for a single i ⟹ Flocking of all agents

Suppose that G(0) is connected. Then

1) G(t) will be connected for all t;

2) All agents asymptotically move with the desired velocity p_r;

3) Collisions among agents are avoided;

Flocking with a virtual leader

Connectivity preserving and only position measurements

$$u_i = -\sum_{j\in N_i} \nabla_{q_i}\psi(\|q_j - q_i\|) - \sum_{j\in N_i} w_{ij}(y_i - y_j) - h_i y_i$$

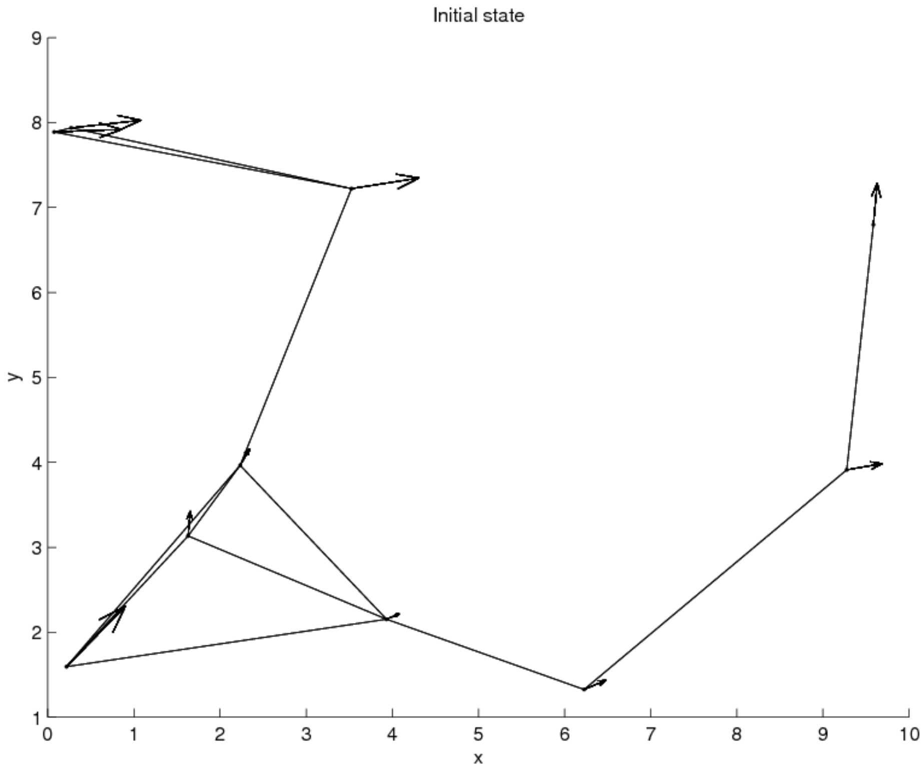
$$y_i = PT\hat{x}_i + P\sum_{j\in N_i} w_{ij}(q_i - q_j) + Ph_i(q_i - q_\gamma)$$

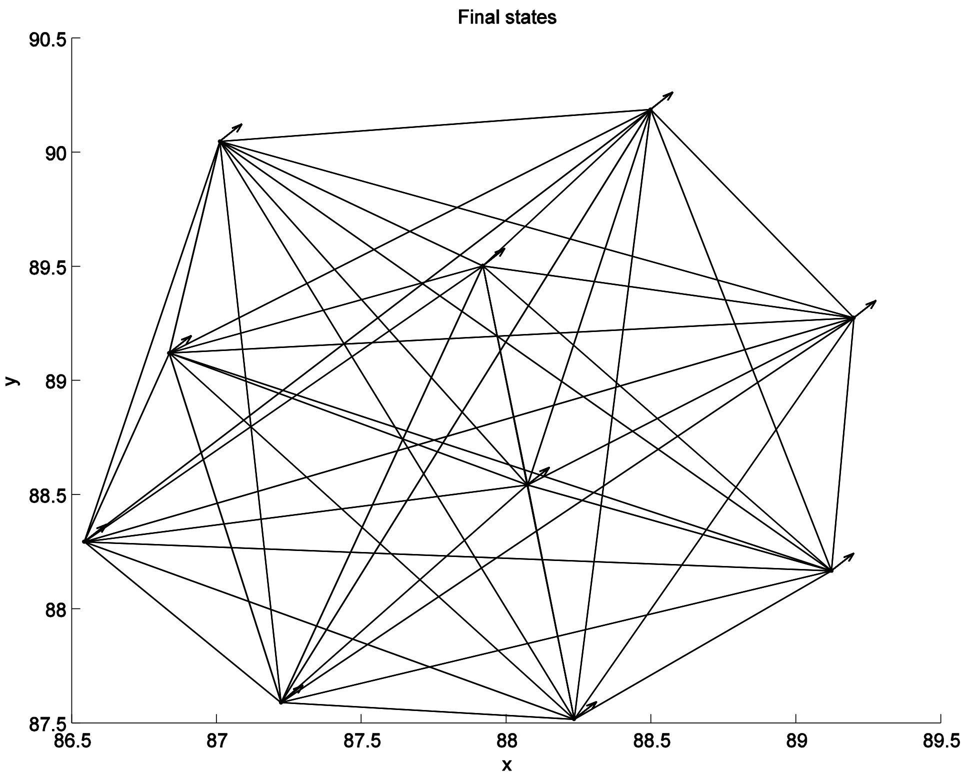
$$\dot{\hat{x}}_i = T\hat{x}_i + \sum_{j\in N_i} w_{ij}(q_i - q_j) + h_i(q_i - q_\gamma)$$

$$T^T P + PT = -Q$$

h_i=1 for a single i ⟹ Flocking of all agents

Flocking with one informed agent





Adaptive Flocking with a Virtual Leader of Multiple Agents Governed by Nonlinear Dynamics

$$\dot{q}_i = p_i,\qquad \dot{p}_i = f(p_i) + u_i, \quad i = 1, 2, \cdots, N,$$

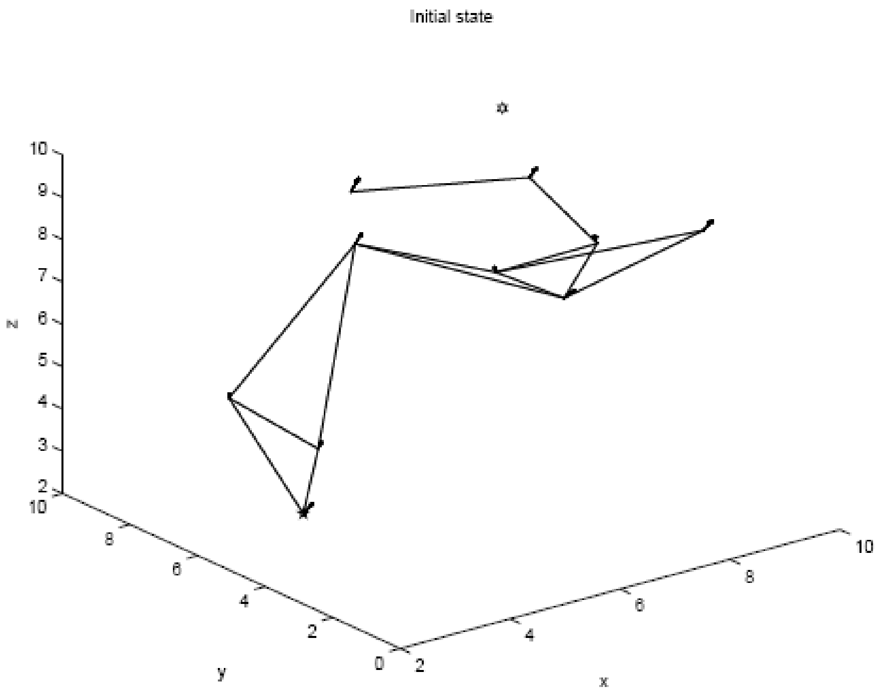
$$\dot{q}_\gamma = p_\gamma,\qquad \dot{p}_\gamma = f(p_\gamma).$$

$$(x-y)^T[f(x)-f(y)] \le (x-y)^T\Delta(x-y), \quad \forall x, y \in \mathbf{R}^n,$$

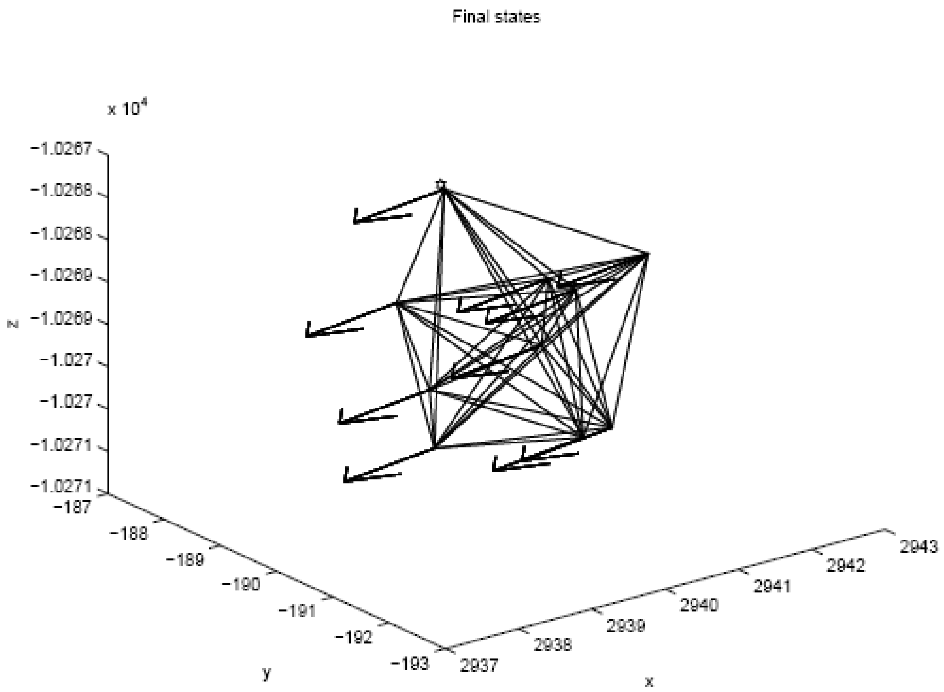
$$\begin{aligned} u_i &= -\sum_{j\in\mathcal{N}_i(t)} \nabla_{q_i}\psi(\|q_{ij}\|) - \sum_{j\in\mathcal{N}_i(t)} m_{ij}(p_i - p_j) \\ &\quad -h_i c_1(q_i - q_\gamma) - h_i c_{2i}(p_i - p_\gamma), \\ \dot{m}_{ij} &= k_{ij}(p_i - p_j)^T(p_i - p_j), \\ \dot{c}_{2i} &= k_i(p_i - p_\gamma)^T(p_i - p_\gamma), \end{aligned}$$

H Su, G Chen, X Wang, Z Lin, Automatica，2011

Simulation

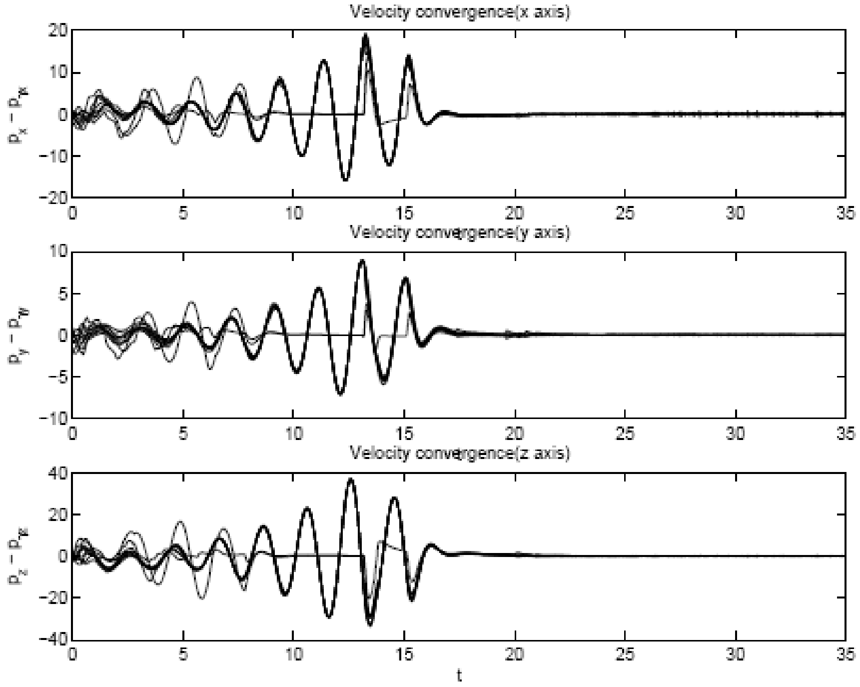


(a)

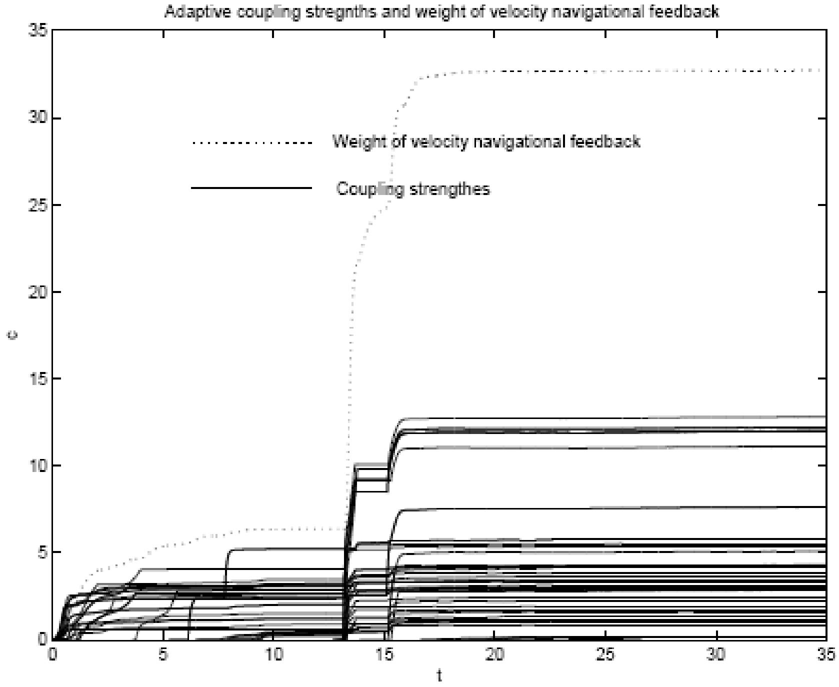


(b)

Simulation



(c)



(d)

Flocking with a Virtual Leader of Multiple Agents Governed by Heterogenous Nonlinear Dynamics

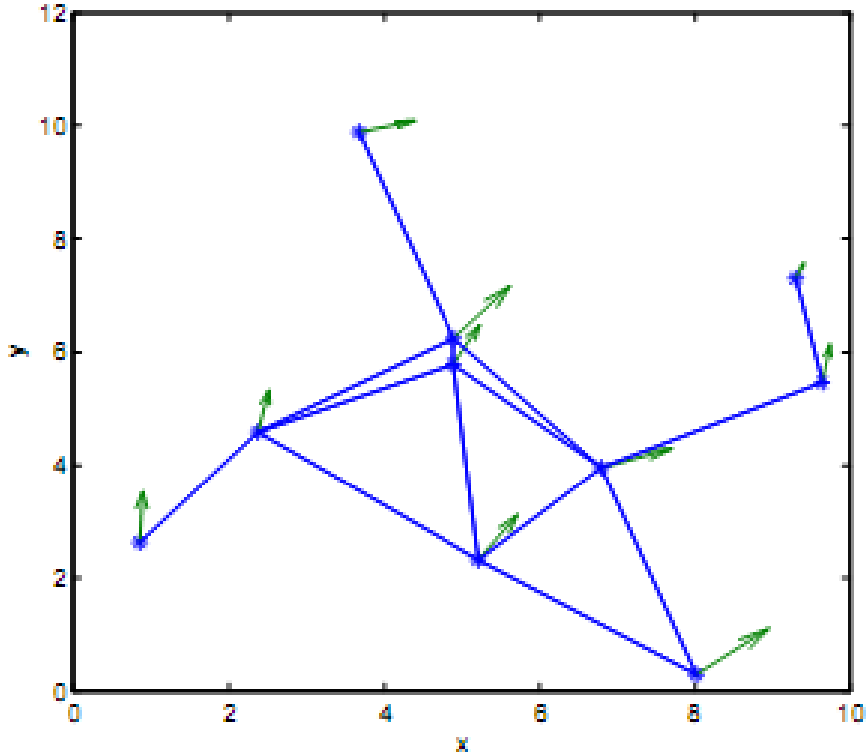
$$\dot{p}_i = v_i, \qquad \dot{p}_\gamma = v_\gamma,$$

$$\dot{v}_i = f_i(v_i) + u_i, \qquad \dot{v}_\gamma = f_\gamma(v_\gamma),$$

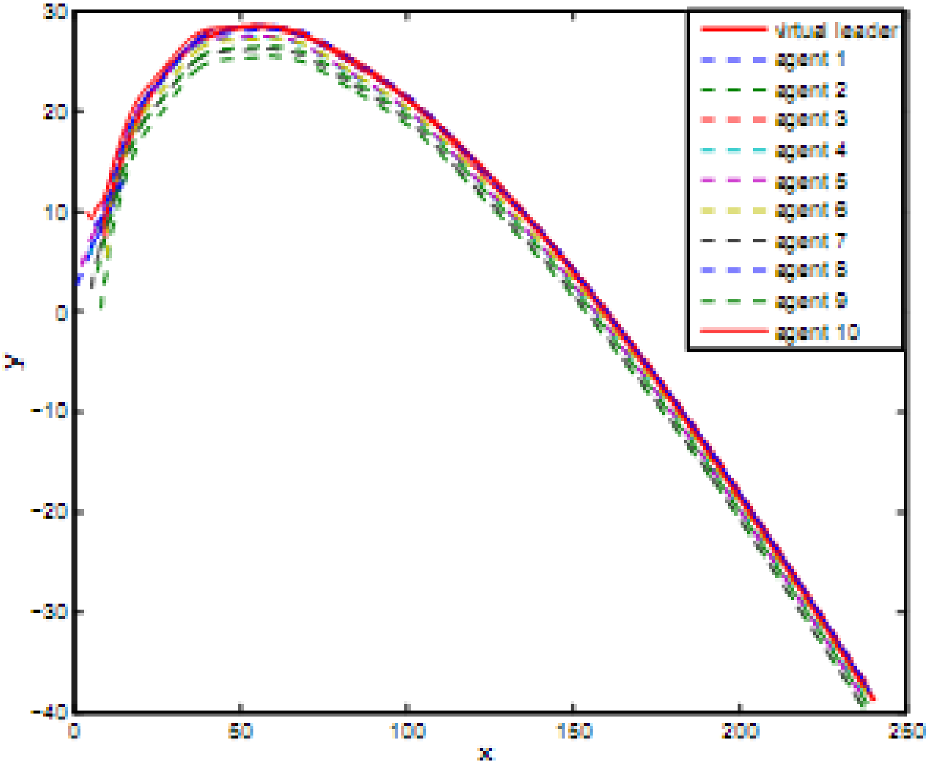
$$\|f_i(v_i) - f_\gamma(v_\gamma)\| \le \ell, \quad \forall v_i, v_\gamma \in \mathbf{R}^n,$$

$$\begin{cases} u_i = \alpha_i + \beta_i + \gamma_i \\ \alpha_i = -\sum_{j\in N_i(t)} \nabla_{p_i}\psi\left(\|p_{ij}\|\right) \\ \beta_i = -h_i c_1(p_i - p_\gamma) \\ \gamma_i = -\rho \sum_{j\in N_i(t)} a_{ij}\left\{\operatorname{sgn}\left[\sum_{k\in N_i(t)} a_{i,k}(v_i - v_k) + h_i(v_i - v_\gamma)\right]\right. \\ \left. -\operatorname{sgn}\left[\sum_{k\in N_j(t)} a_{j,k}(v_j - v_k) + h_j(v_j - v_\gamma)\right]\right\} \end{cases}$$

Simulation

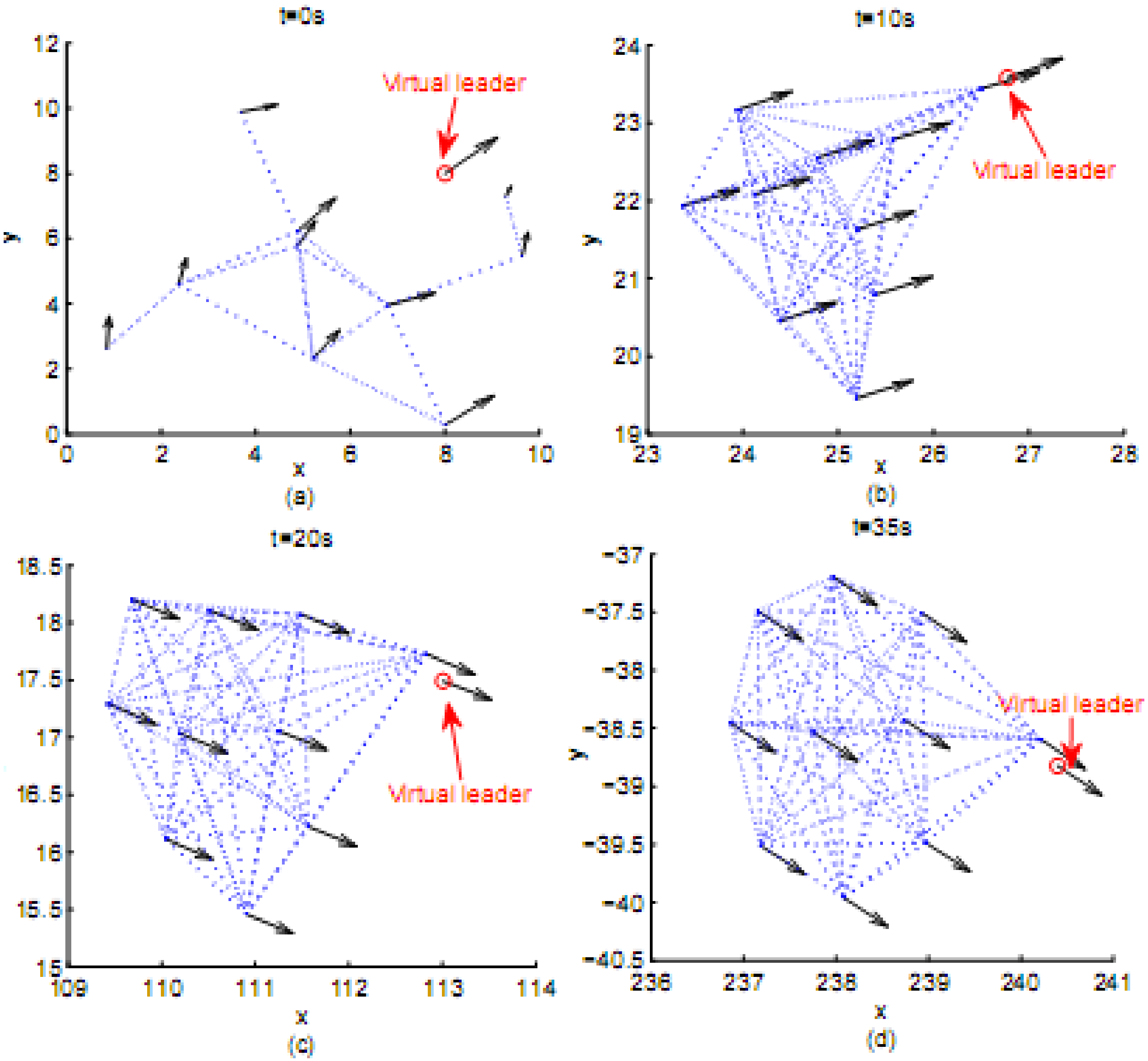


(a)

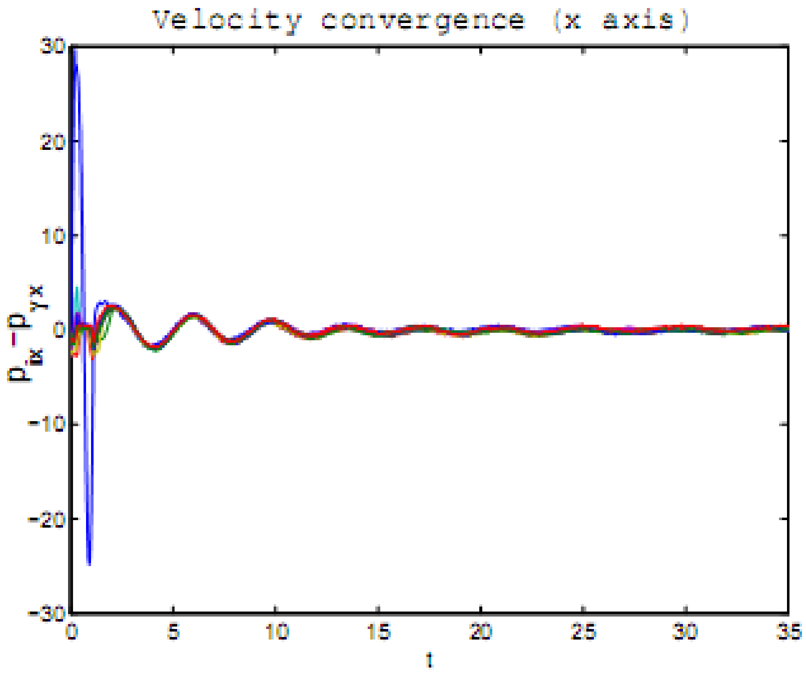


(b)

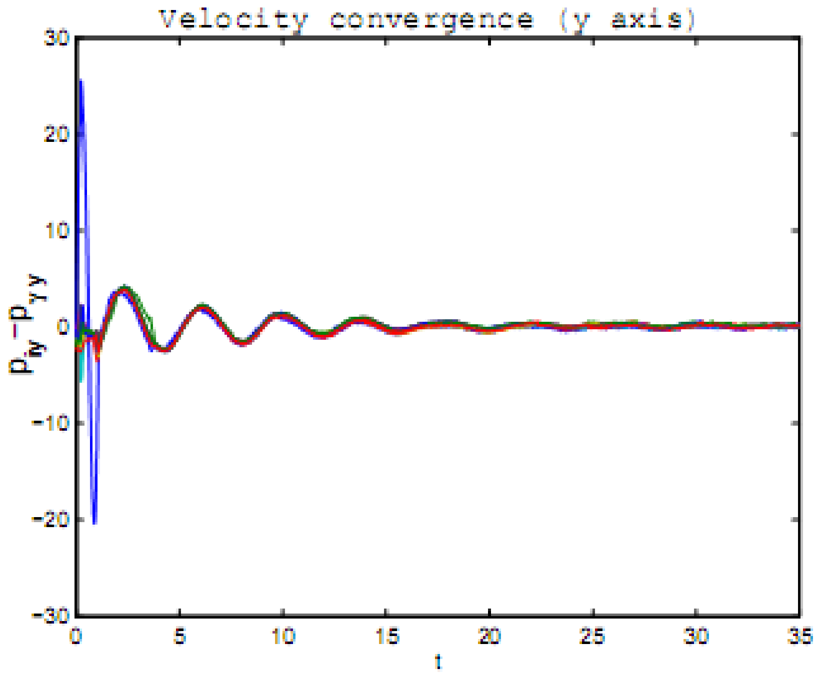
Simulation


t=0s
Virtual leader
y
x
(a)
t=10s
Virtual leader
y
x
(b)
t=20s
Virtual leader
x
(c)
t=35s
Virtual leader
y
x
(d)

Simulation



(a)



(b)

Conclusion

Flocking without connectivity preserving

- A minority of informed agents can guarantee flocking of a majority of agents (pinning control)

Flocking with connectivity preserving (New potential fun. +neighboring)

- position measurements (Observer)
- Nonlinear intrinsinc dynamics (Adaptive)
- A single informed agent can guarantee flocking of all agents (pinning control)

Thanks !