Pinning Control:

from Synchronization to Social Learning

Xiaofan Wang

xfwang@sjtu.edu.cn

OUR RESEARCH ON PINNING CONTROL

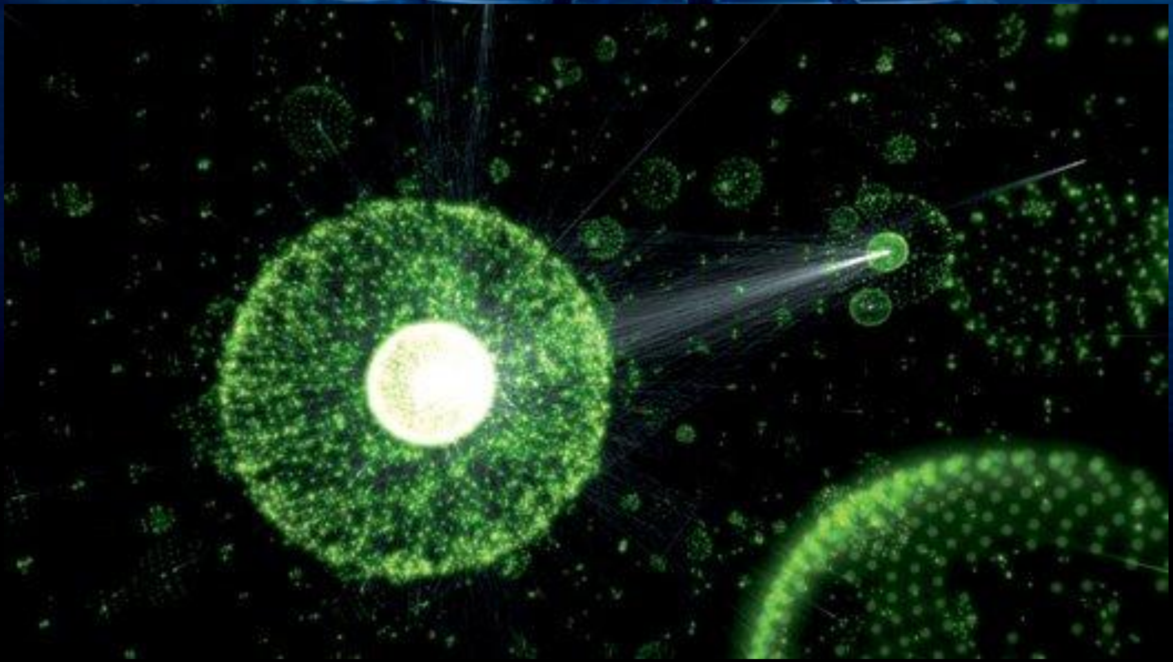
Pinning Controlled

- Synchronization, 2002
- Swarming, 2006
- Flocking, 2009
- Social Learning, 2011

Pinning Control: Motivations

Directly control every node in a huge-scale network

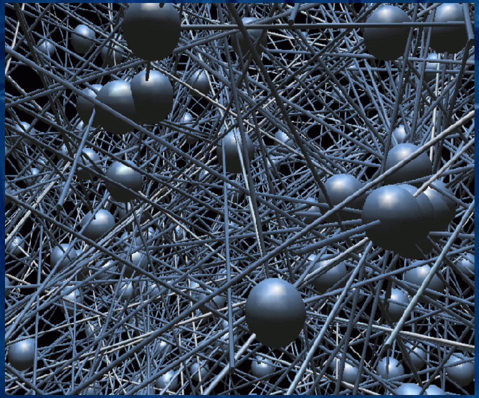
may be practically impossible & unnecessary!



Pinning Control: Basic Problems

- **Feasibility:** Can the goal of control be achieved by only directly control a fraction of nodes?

--- Control Science



- **Efficiency:** How to select the nodes to be controlled so that the goal can be achieved with a low cost?

--- Network Science

Case I: Pinning Controlled Synchronization

$$\begin{cases} \dot{x}_k = f(x_k) + c\sum_{j=1}^{N} a_{kj}\Gamma x_j + u_k, & k = 1, 2, \cdots, l \\ \dot{x}_k = f(x_k) + c\sum_{j=1}^{N} a_{kj}\Gamma x_j, & k = l+1, l+2, \cdots, N \end{cases}$$

Objective: Stabilize onto a homogeneous equilibrium

$$x_1(t) = x_2(t) = \cdots = x_N(t) = \bar{x}$$

Linear Error Feedback: $u_k = -cd(x_k - \bar{x})$

Wang & Chen, 2002

Feasibility: Stability Criterion

$$\begin{cases} \dot{x}_k = f(x_k) + c\sum_{j=1}^{N} a_{kj}\Gamma x_j - cd\Gamma\left(x_k - \overline{x}\right), & k = 1, 2, \cdots, l \\ \dot{x}_k = f(x_k) + c\sum_{j=1}^{N} a_{kj}\Gamma x_j, & k = l+1, l+2, \cdots, N \end{cases}$$

$$B = A - D$$

$$D = diag(d_1, d_2, \cdots, d_N)$$

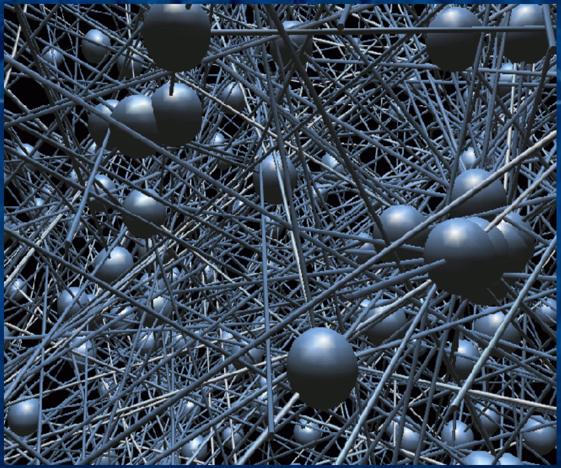
$$c \geq \left|\rho / \lambda_1\right|$$

Wang & Chen, 2002

Efficiency of Pinning Control:

Two Pinning Schemes

- Specifically pinning scheme

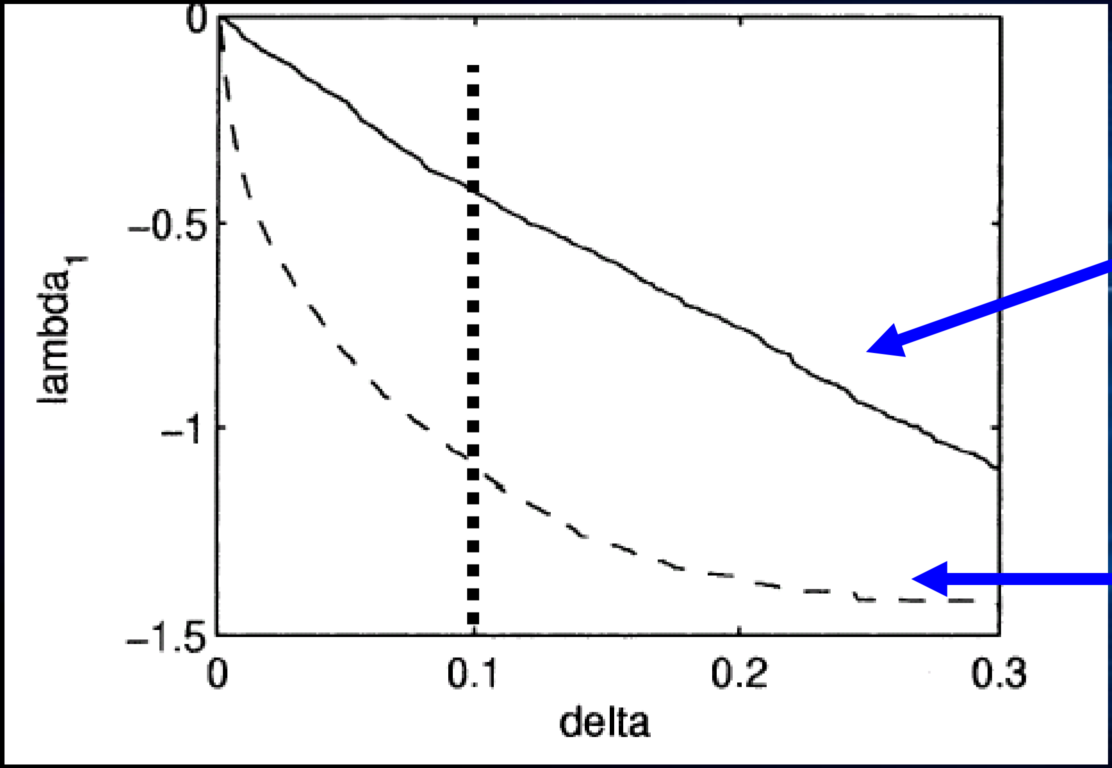


- Randomly pinning scheme

Wang & Chen, 2002

Pinning Control of a BA Scale Free Network

$$c \geq \left| \rho / \lambda_1 \right|$$



- Randomly
- Specifically

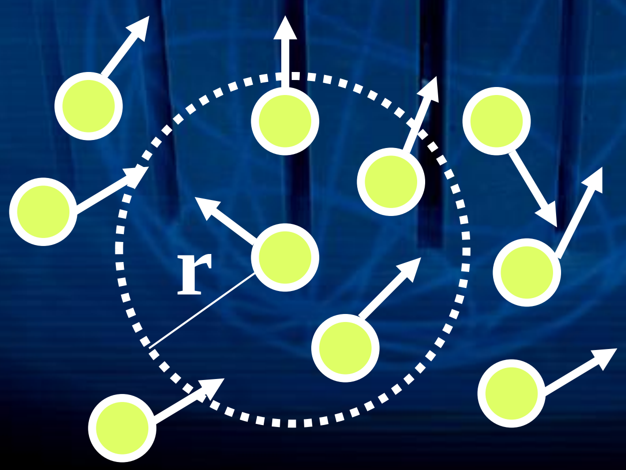
- 3000 nodes and about 9000 edges

Wang & Chen, 2002

Case II: A Modified Vicsek Model

Average direction $\theta_i(k) = <\theta_i(k-1)>_r + \Delta\theta$

Constant speed $v_i(k) = v_0$



Vicsek, et al., Phys. Rev. Lett., 75 (6), 1226, 1995

Different agents may have different influencing abilities

$$P(r) \sim r^{-\gamma} \quad \gamma \in [2, \infty)$$

$\gamma \uparrow$

Heterogeneity $\downarrow$

Vicsek model:

$$\gamma \to \infty$$

$$\theta(t+1) = <\theta(t)>_r$$

W. Yang, L Cao & X Wang, PRE, 2006

Simulation Results

Model: $\theta(t+1) = <\theta(t)>_r$ $\quad P(r) \sim r^{-\gamma} \quad \gamma \in [2, \infty)$

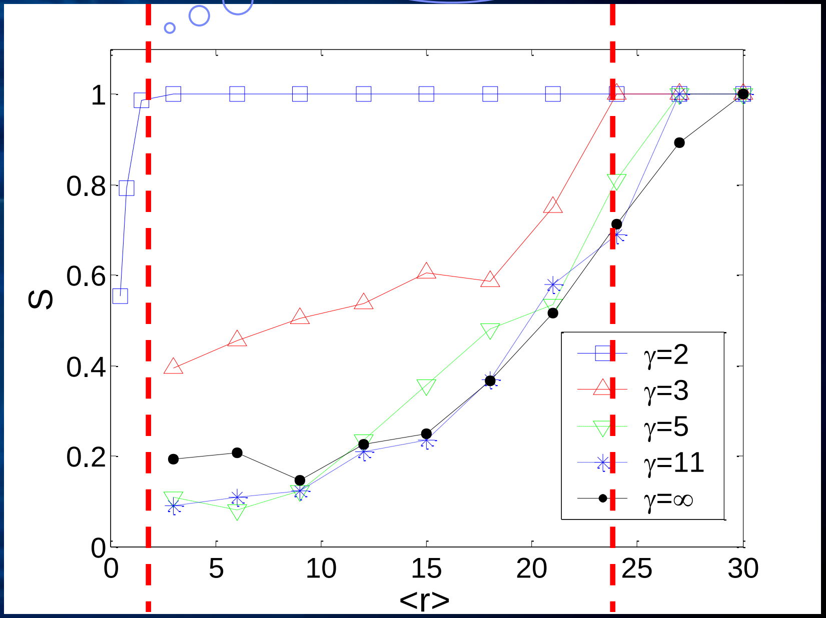
S: largest cluster

$\bar{r}(\gamma)$: S=1 (global syn.)

$\bar{r}(\gamma)$

$\gamma \uparrow, \quad \bar{r}(\gamma) \uparrow$

Heterogeneity $\downarrow$

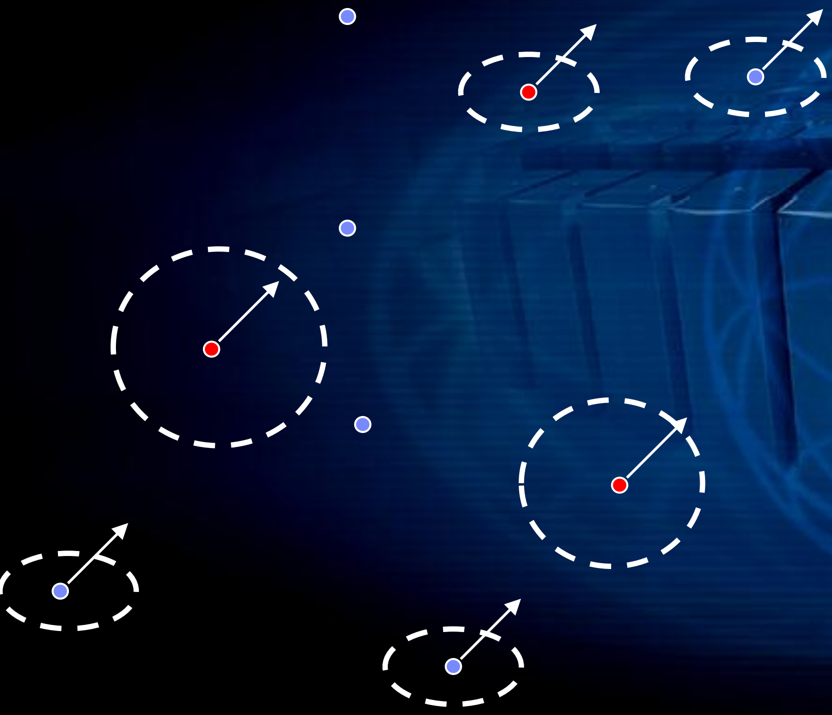


$L = 50 \quad n = 1250 \quad v = 0.1$

Heterogeneity benefits consensus!

Pinning Control Scheme

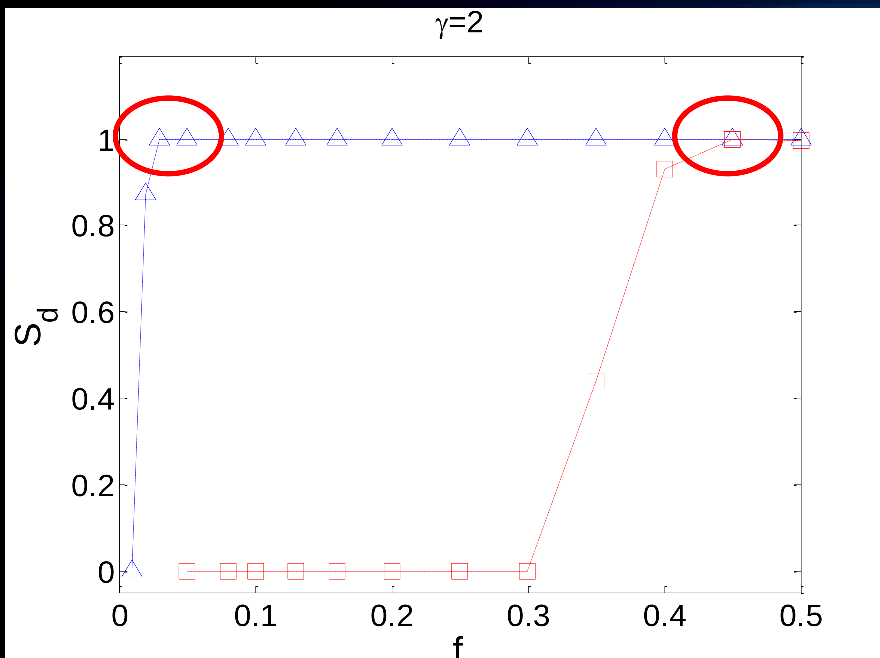
❑**Goal:** achieve direction consensus in an influence network with a desired direction by pinning a fraction of agents.

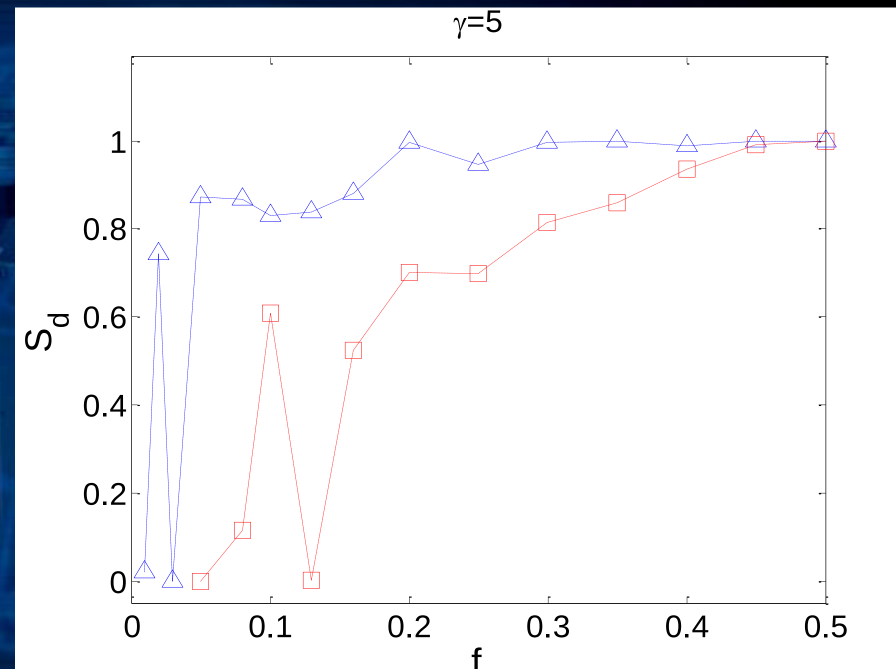


- Specifically pinning control
  - control f% agents with largest radii
- Randomly pinning control
  - control f% randomly chosen agents
- fixed same direction

The Largest Desired Cluster S_d

$$P(r) \sim r^{-\gamma} \quad \gamma \in [2, \infty)$$



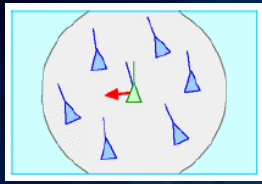


Heterogeneous **Homogeneous**

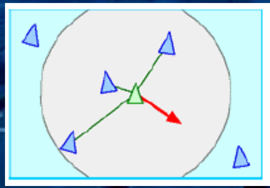
$$L = 50 \quad N = 1250 \;\; v = 0.1 \;\; < r >= 6$$

Case III: Pinning Controlled Flocking

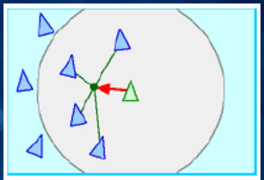
- Alignment



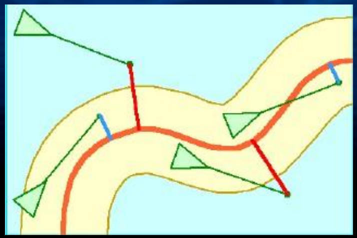
- Separation



- Cohesion



- Goal Seeking (Tracking)



Flocking with a Virtual Leader

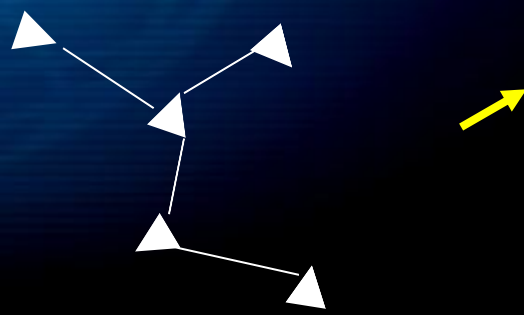
$$\dot{q}_i = p_i \qquad \dot{p}_i = u_i$$

$$\begin{cases} \dot{q}_\gamma = p_\gamma \\ \dot{p}_\gamma = f_\gamma(q_\gamma, p_\gamma) \end{cases}$$

$$u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|)$$ **Separation & Cohesion**

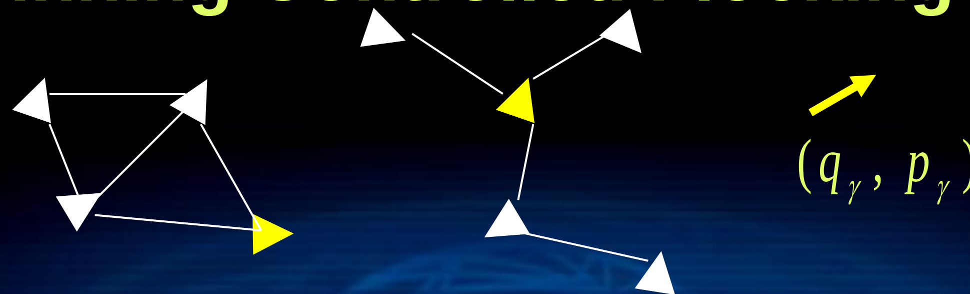
$$-\sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$$ **Alignment**

$$-c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma)$$ **Tracking**



Olfati-Saber, IEEE Trans AC，2006

Pinning Controlled Flocking



Uninformed agent

$$u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|) - \sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$$

Separation & Cohesion　　Alignment

Informed agent

$$u_i = -\sum_{j \in N_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|) - \sum_{j \in N_i(t)} w_{ij}(p_i - p_j)$$

$$-c_1(q_i - q_\gamma) - c_2(p_i - p_\gamma)$$ Tracking

H Su, X Wang & Z Lin (2009). *IEEE Trans. Automatic Control*

Case IV: Pinning Controlled Social Learning

人肉搜索为何几乎百发百中？

教育医疗等却为何如此难以形成最佳共识？

Social Learning on Networks

Private Observation

COmmunication → Update → Beliefs

1. Consensus ?
2. Best Consensus ?
3. Controllabe ?

Case study: Who is singing?

State space $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$

True state $\theta^* \in \Theta$

Private signal s_t^i

Likelihood function $\sum_s l_i(s \mid \theta) = 1$

Network structure

Belief $\mu_{i,t}(\theta)$

Bayesian+ Consensus

$$\mu_{i,t+1}(\theta) = a_{ii}\,\frac{l_i(s_{t+1}^i \mid \theta)\mu_{i,t}(\theta)}{m_{i,t}(s_{t+1}^i)} + \sum_{j \in N_i} a_{ij}\,\mu_{j,t}(\theta)$$

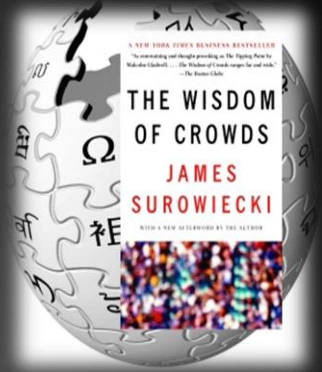
Bayesian

Consensus

$$m_{i,t}(s_{t+1}^i) = \sum_{\theta \in \Theta} l_i(s_{t+1}^i \mid \theta)\mu_{i,t}(\theta)$$

[Jadbbaie, Sandroni, and Tahbaz-saleh 2010]

The Wisdom of Crowds



(a) The social network is **strongly connected**;

(b) All agents have **strictly positive self-reliances**;

(c) There exists an agent with **positive prior belief** on the true state;

(d) There is no other state that is **observationally equivalent** to the true state from the point of all agents in the network.

Social learning with uninformed agents

Suppose that

(a) The social network is strongly connected;

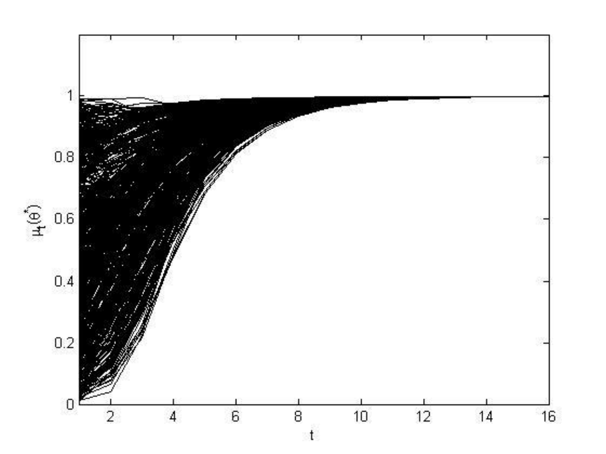
(b) There exists at least one informed agent and all self-reliance of informed agents are strictly positive;

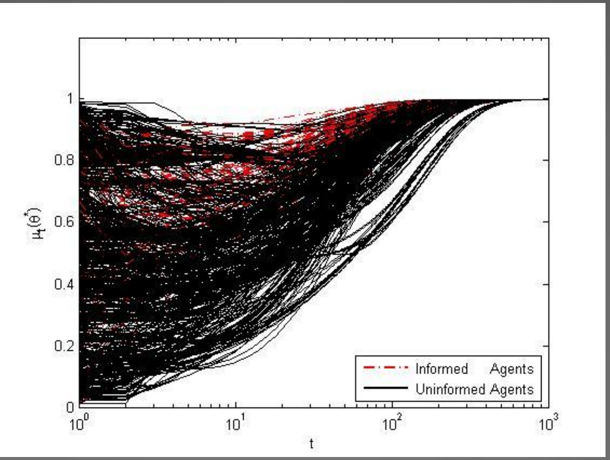
(c) There exists at least one agent with positive prior belief on the true state;

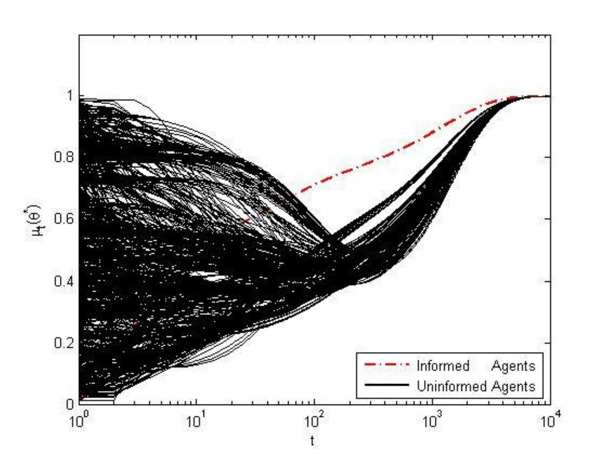
(d) There is no state that is observationally equivalent to the true state from the point of all informed agents in the network.

Social learning with uninformed agents

In a component of Wealink







l=1157

l=100

l=1

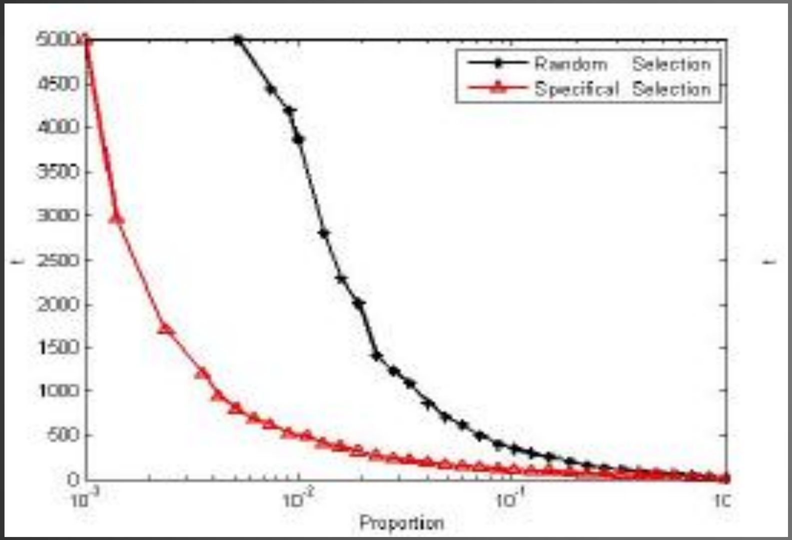
N=1157, two states, two signals {H, T}

Prior beliefs: uniform distribution in [0,1], a_{ii}=0.5

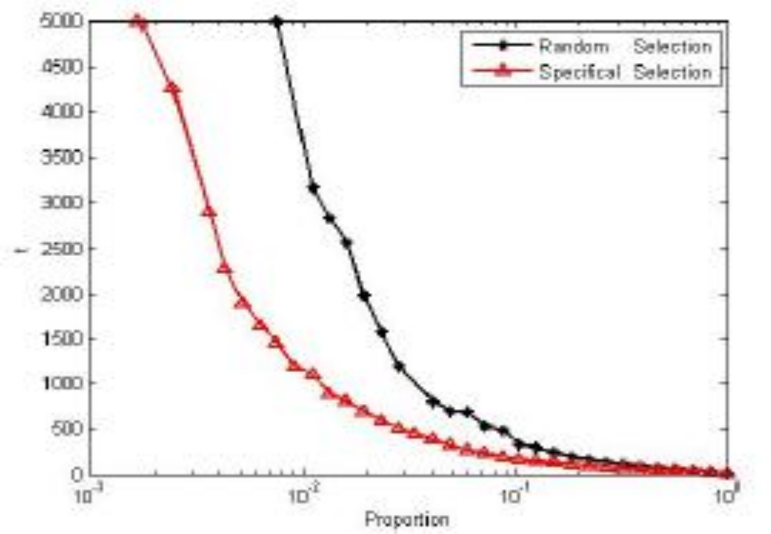
Social learning with uninformed agents

In a power-law network with tunable exponent

$\gamma = 2.1$ *Heterogeneous*



$\gamma = 10$ *Homogeneous*

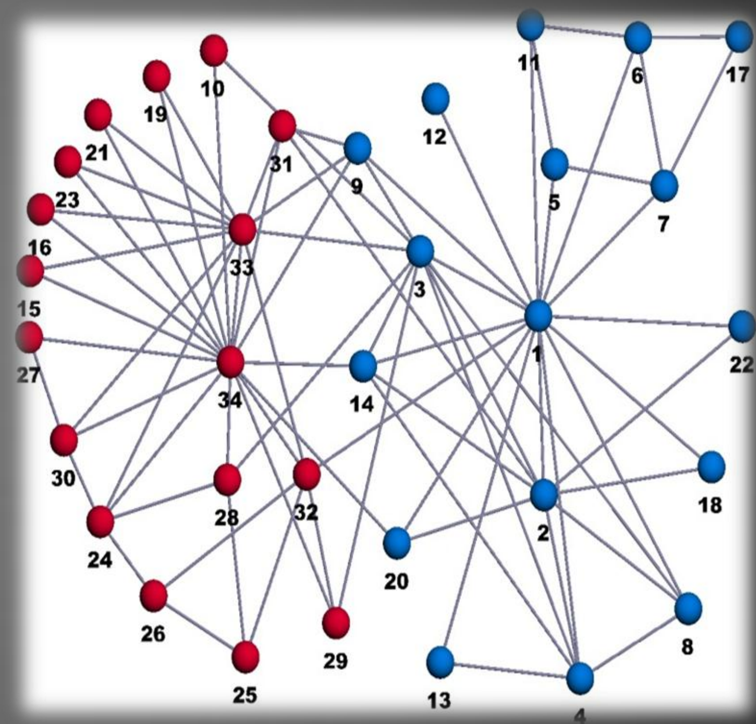


N=1000, two states, two signals {H, T}

Prior beliefs: uniform distribution in [0,1], a_{ii}=0.5

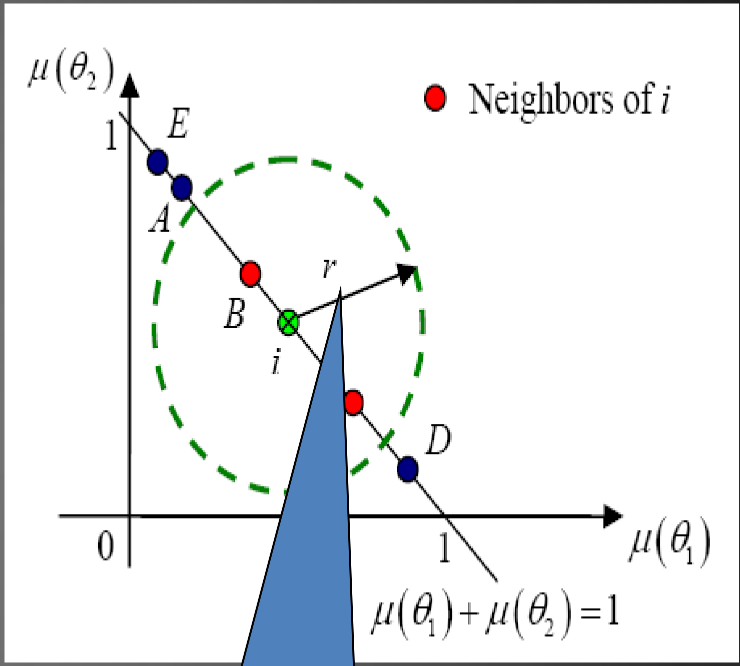
Similarity Breeds Connection:

Homophily Principle in Sociology



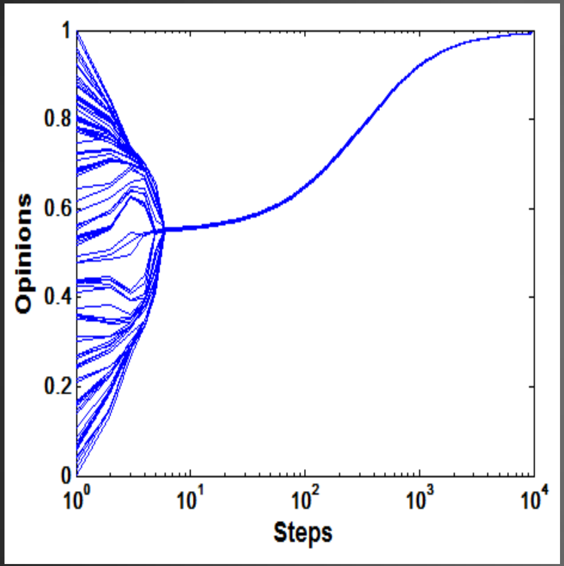
Social learning with similarity-based communication

$$\mu_{i,t+1}(\theta)=\frac{1}{|N_i(t)|}\left(\mu_{i,t}(\theta)\frac{p\left(s_{t+1}^i\,|\,\theta\right)}{m_{i,t}\left(s_{t+1}^i\right)}+\sum_{j\in N_i(t)}\mu_{j,t}(\theta)\right)$$

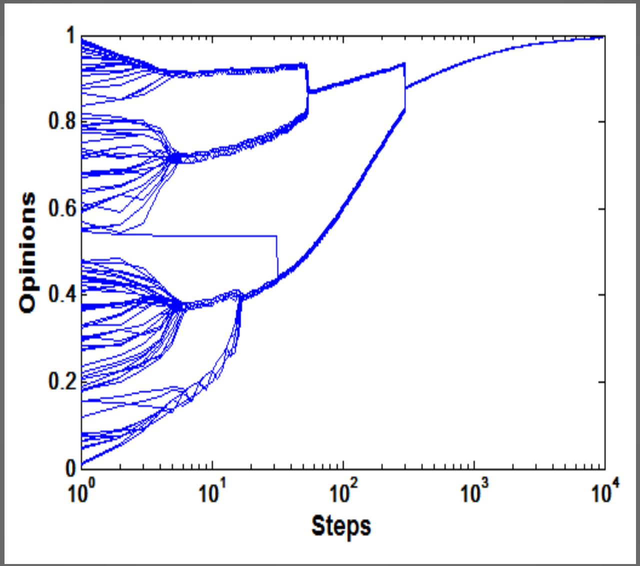


Confidence radius

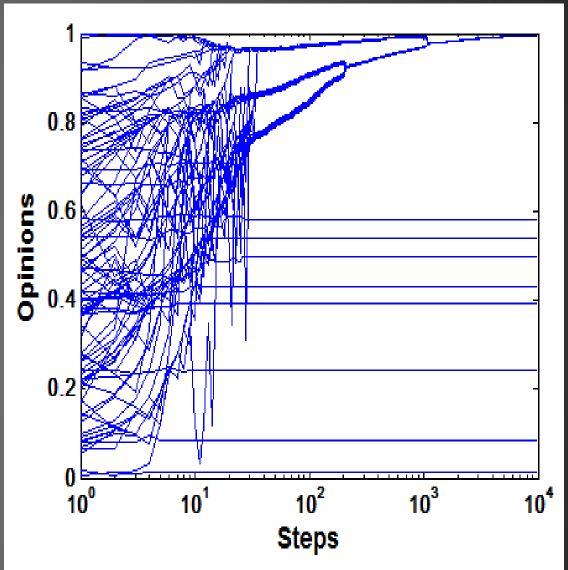
Social Learning with Bounded Confidence



r = 0.3

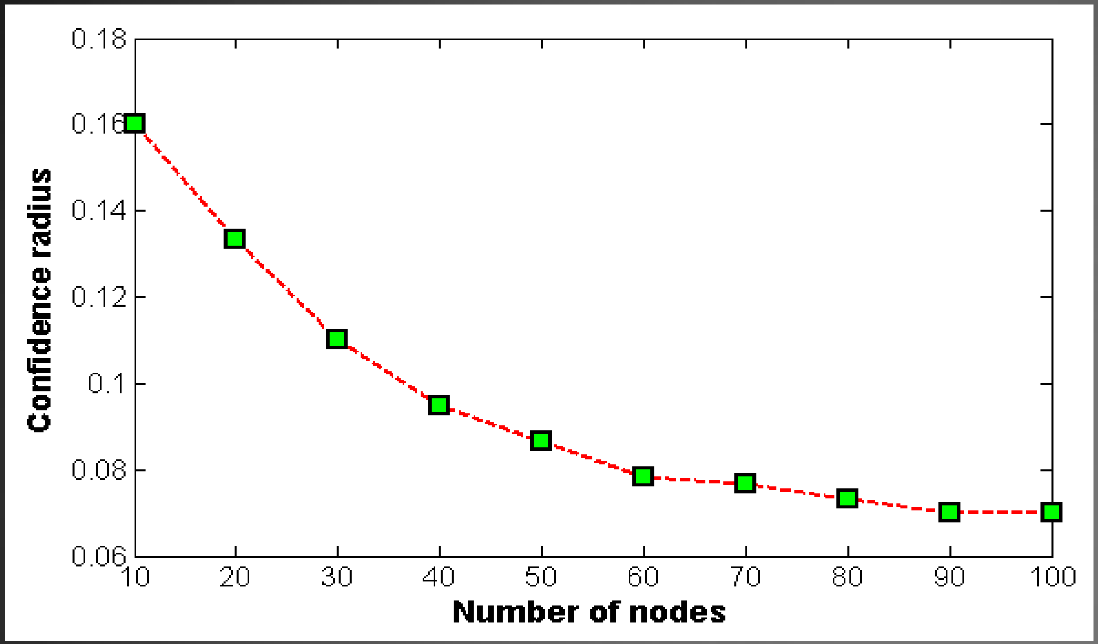


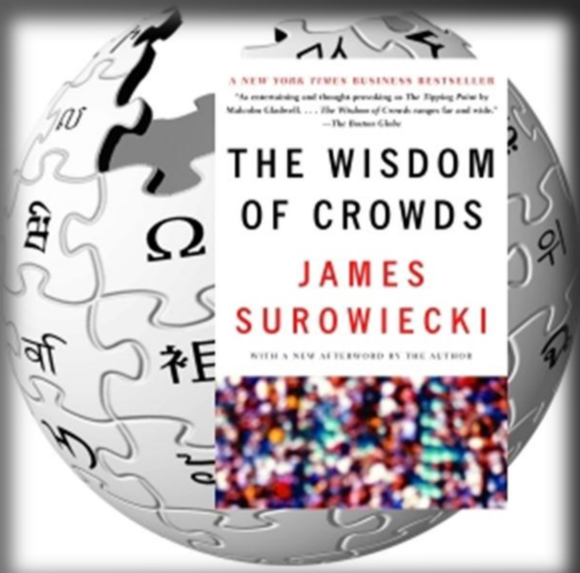
r = 0.1



r = 0.02

The Wisdom of Crowds





The larger the group， the smaller the confidence radius needed for asymptotic learning.

Social Learning on Networks

Private Observation

COmmunication → Update → Beliefs

1. Consensus ?
2. Best Consensus ?
3. Controllabe ?

谢谢！