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Multi-Agent Systems: Nexus of All Realities in Systems and Control

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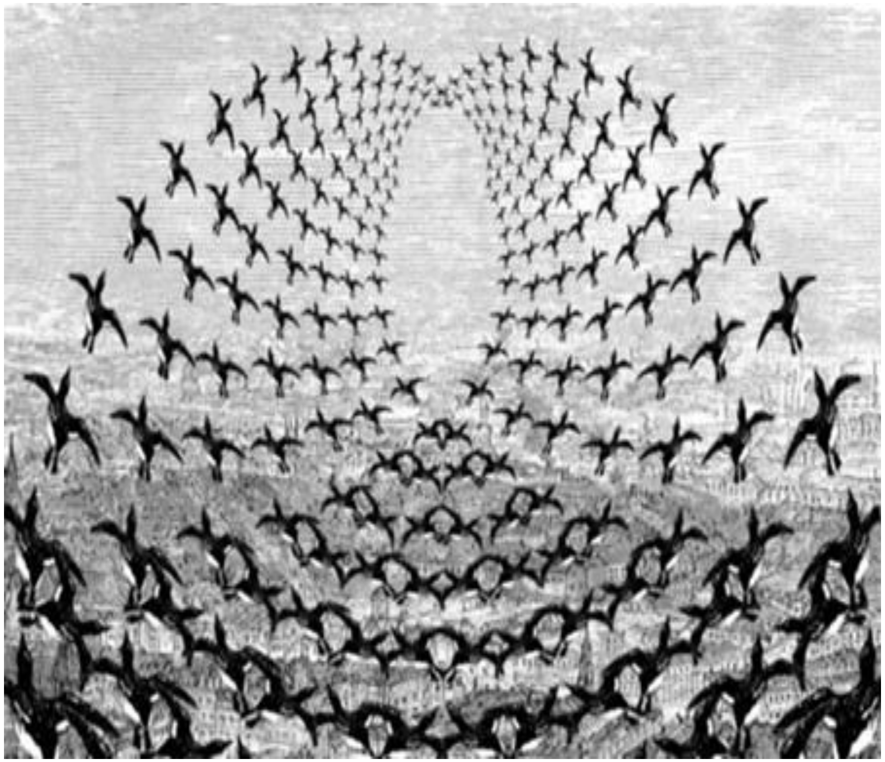
Outline

- **Introduction**
- **Modeling, Analysis, Control of MAS: An Overview**
- **Consensus of MAS: Several Typical Cases**
- **Conclusions**

Introduction

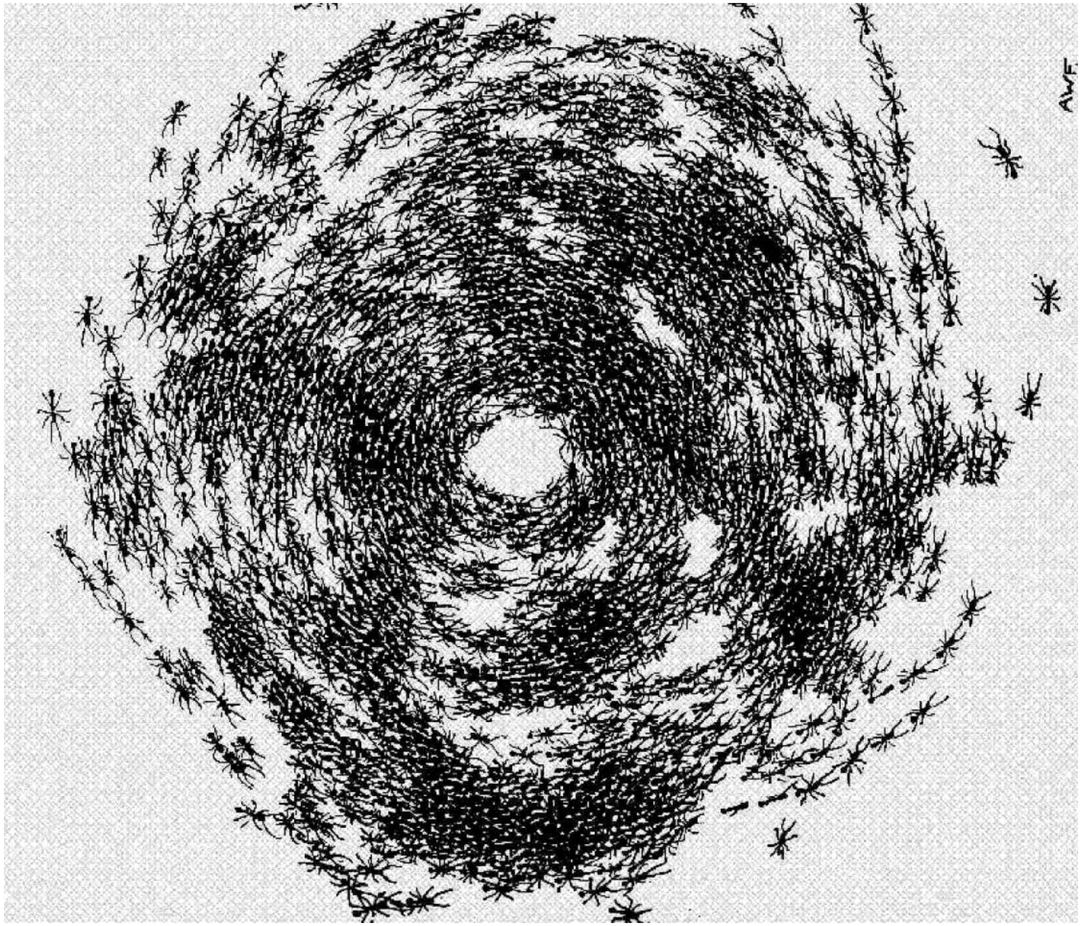
- Some Examples
- What is Multi-Agent Systems (MAS)?
- Main Characteristics of MAS

Examples: Bird Flock

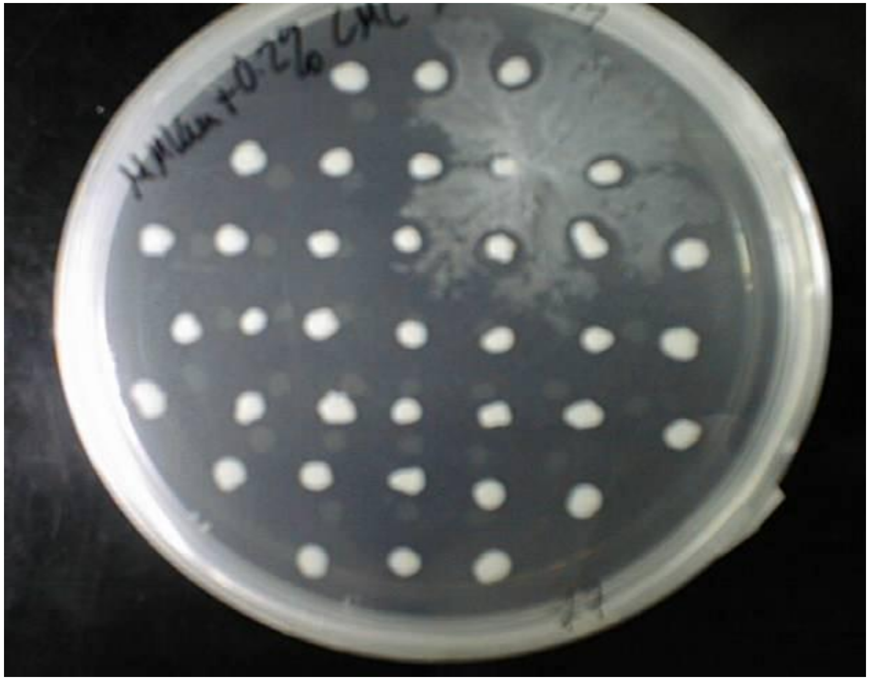


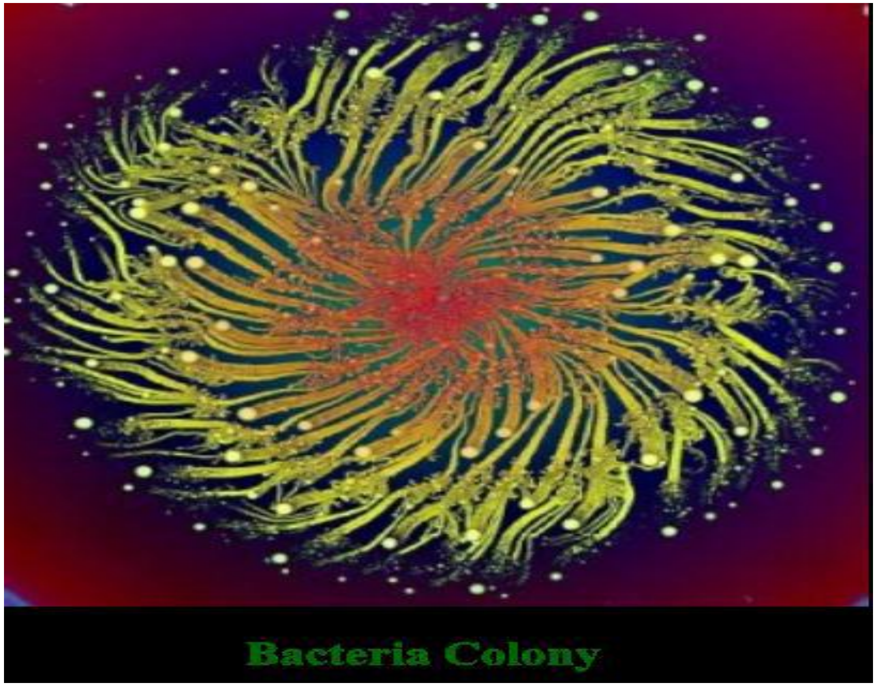
Examples: Fish Flock

Examples: Rotating Ants Mill



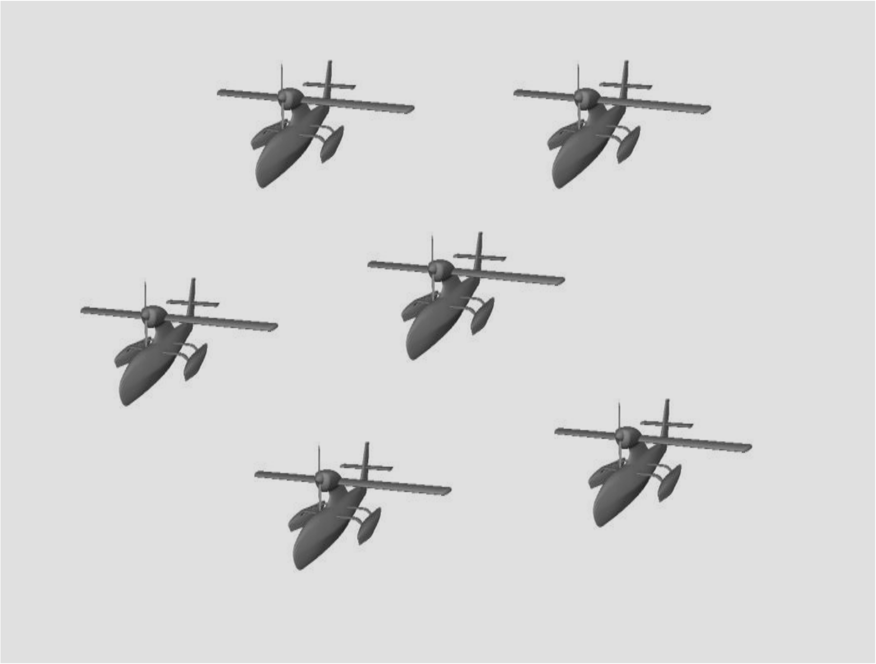
Examples: Bacteria Group





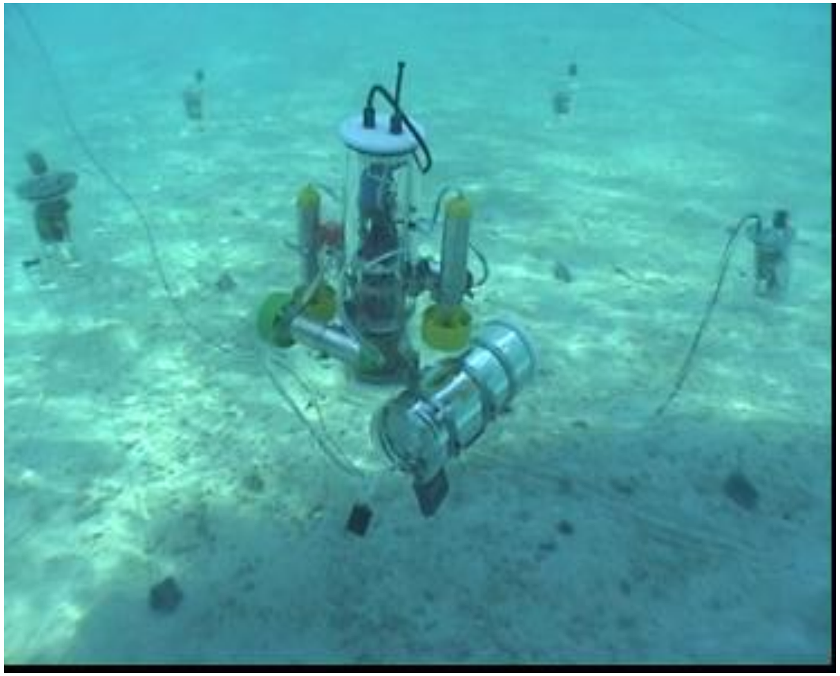
Examples: Social Systems

Examples: Formation Control



- The problem: Maintaining a formation in 2D or 3D
- Autonomous agent

Examples: Group Robots



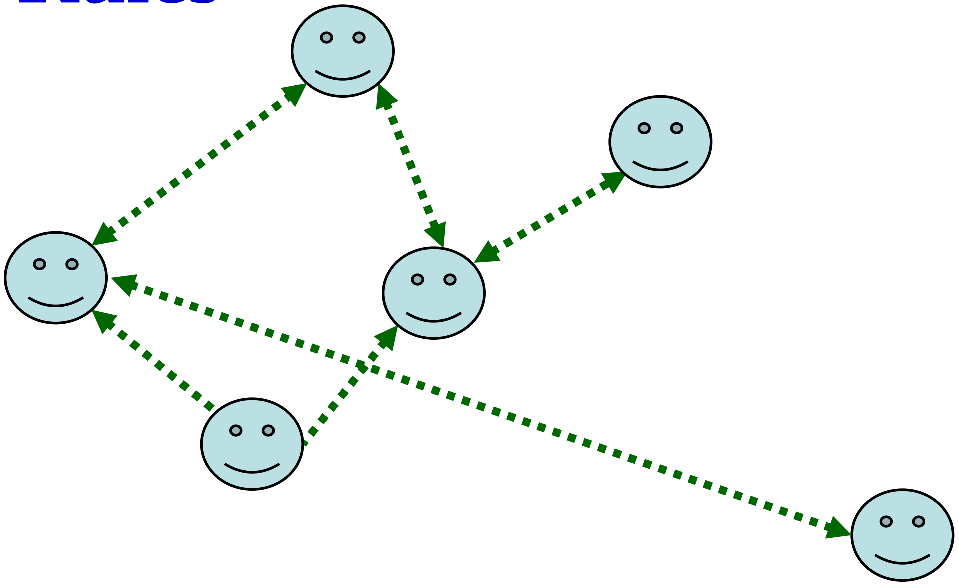
What is Multi-Agent Systems (MAS)?

- **Agents**

  Insect, bird, fish, people, robot, …

  node, individual, particle, …

- **Local Rules**



Main Characteristics of MAS

- **Autonomous/Self-Driven Agents**
- **Distributed Region**
- **Local Interactions**
- **Dynamic Neighbors**
- **Various Connections**
- **Conformable Behaviors**

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- Conclusions

Modeling, Analysis, and Control of MAS: An Overview

- **Modeling**

  —Vicsek, Boid, Couzin-Levin, Complex Dynamical Networks

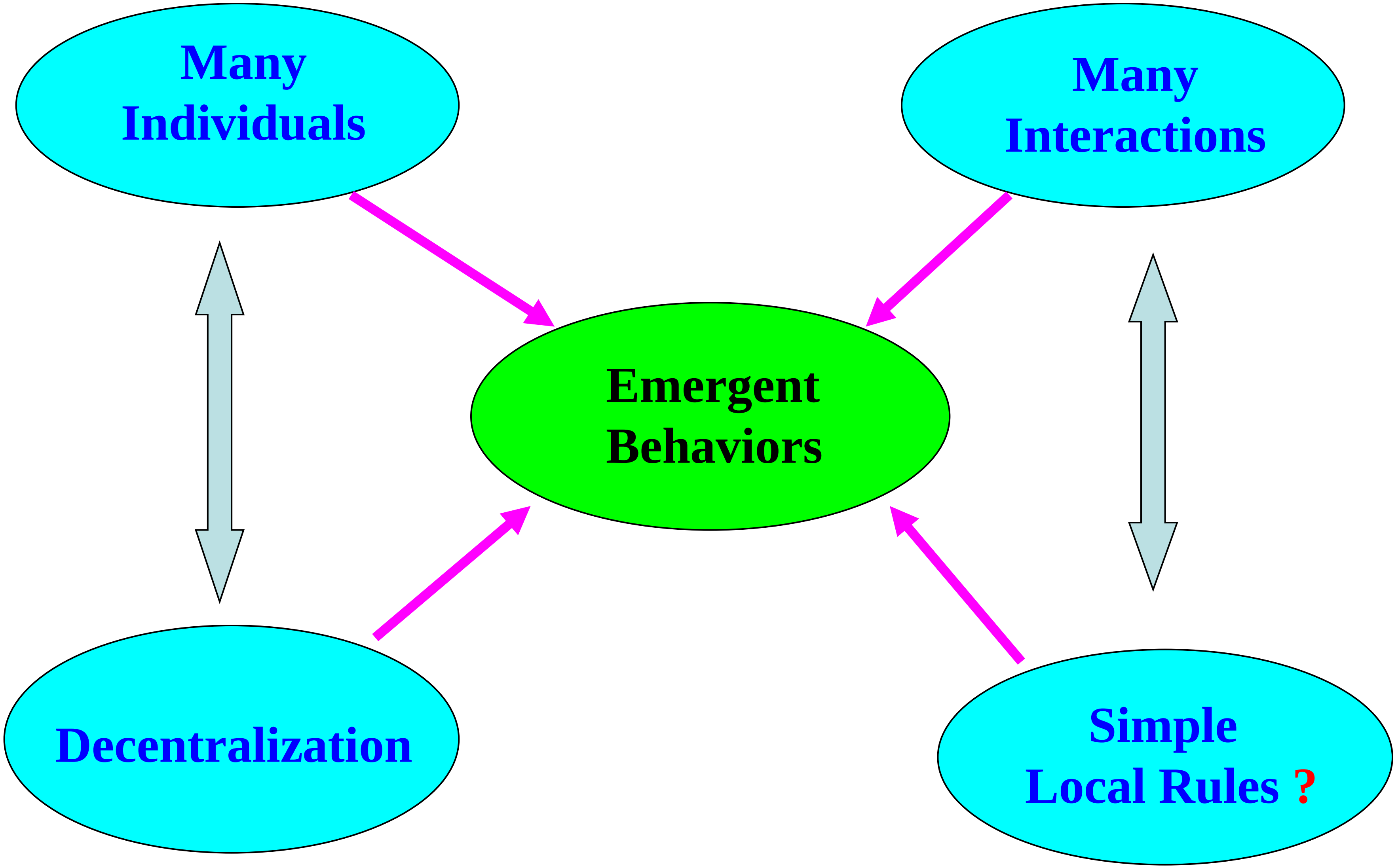
- **Analysis**

  —Consensus, Convergence, Adaptation, Decision-Making, …

- **Control**

  —Leader-Follower, Pinning Control, Formation Control, Cooperation, Intervention, …

A Unifying Framework of MAS



Collective Behaviors: Modeling, Analysis and Control

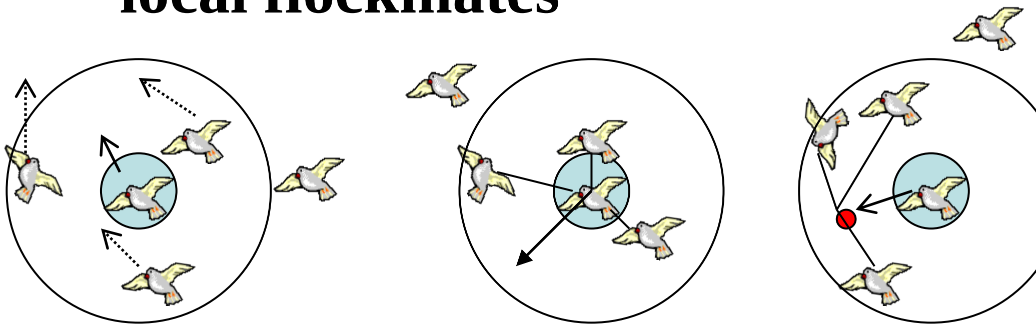
Several Representative Models: Boids Flocking Model (1987)

Three Basic Local Rules: (The First Model)

Alignment: Steer to move toward the average heading of local flockmates

Separation: Steer to avoid crowding local flockmates

Cohesion: Steer to move toward the average position of local flockmates



C. W. Reynolds, Flocks, herd, and schools: A distributed behavioral model, Computer Graphics, 1987, 21(4): 15-24.

Several Representative Models:
Vicsek Particles Model (1995)

One Basic Local Rule: Alignment (The Simplest Model)

- **Position:**

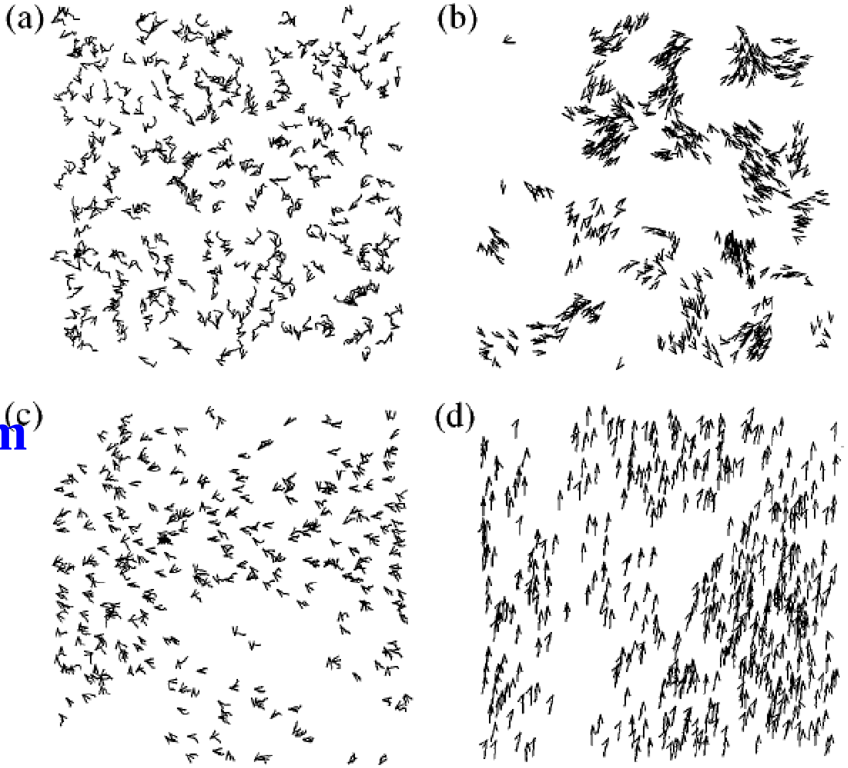
$$x_i(t+1) = x_i(t) + v_i(t)\Delta t$$

- **Heading:**

$$\theta(t+1) =< \theta(t) >_r + \Delta\theta$$

T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, O. Sochet, Novel type of phase transition in a system of self-driven particles, Phys. Rev. Lett., 1995, 75 (6): 1226.

Several Representative Models:
Vicsek Particles Model (1995)



(a) Initial: Random positions/velocities $(\rho = 6.12)$

(b) Low density/noise: grouped, random $(\rho = 0.48)$

(c) High density/noise: correlated, random $(\rho = 6.12)$

(d) High density / Low noise $(\rho = 12)$

→ ordered motion

T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, O. Sochet, Novel type of phase transition in a system of self-driven particles, Phys. Rev. Lett., 1995, 75 (6): 1226.

Several Representative Models:
Couzin-Levin Model (2005)

Local Rules: Separation, Cohesion, Alignment

Updating rules: (Information)

1) If $r(i,j) \leq \alpha$, then

$$d_i(t+\Delta t) = -\sum_{j\neq i} \frac{c_j(t)-c_i(t)}{|c_j(t)-c_i(t)|}$$

2) If $\alpha < r(i,j) \leq \rho$, then

$$d_i(t+\Delta t) = \sum_{j\neq i} \frac{c_j(t)-c_i(t)}{|c_j(t)-c_i(t)|} + \sum_{j=1} \frac{v_j(t)}{|v_j(t)|}$$

3) Consider information direction

$$d_i'(t+\Delta t) = \frac{\hat{d}_i(t+\Delta t)+wg_i}{|\hat{d}_i(t+\Delta t)+wg_i|}$$

I. D. Couzin, J. Krause, N. R. Franks, & S. M. Levin, Effective leadership and decision-making in animal groups on the Move, Nature, 2005, 433: 513-516.

Analysis of MAS: Main Approaches

- **Numerical Simulations**

  ——Simulation Platform, Mathematical or Computer Model

- **Theoretical Analysis**

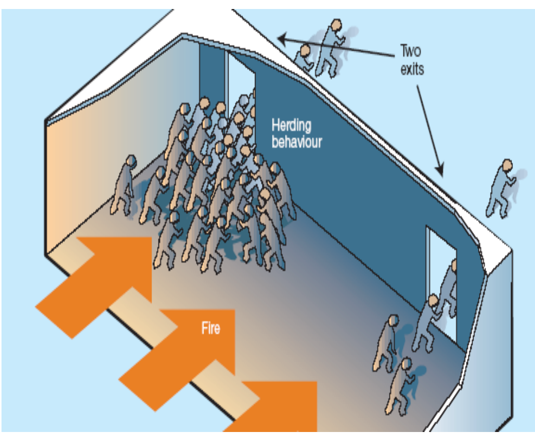
  ——Lyapunov Function, Eigenvalue Computation, Convex Analysis, Stochastic Approximation, Graph Theory, …

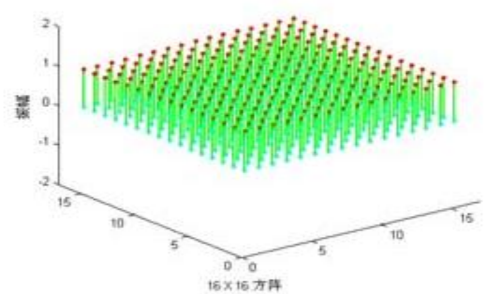
- **Experimental Observation**

  ——Fish, Locust, Bird, Ants, …

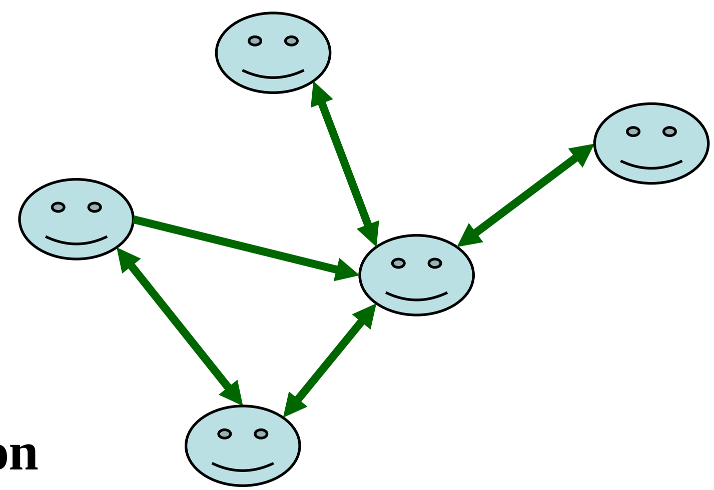
Numerical Simulations

- Learning
- Adaptation
- Cooperation
- Competition
- ……



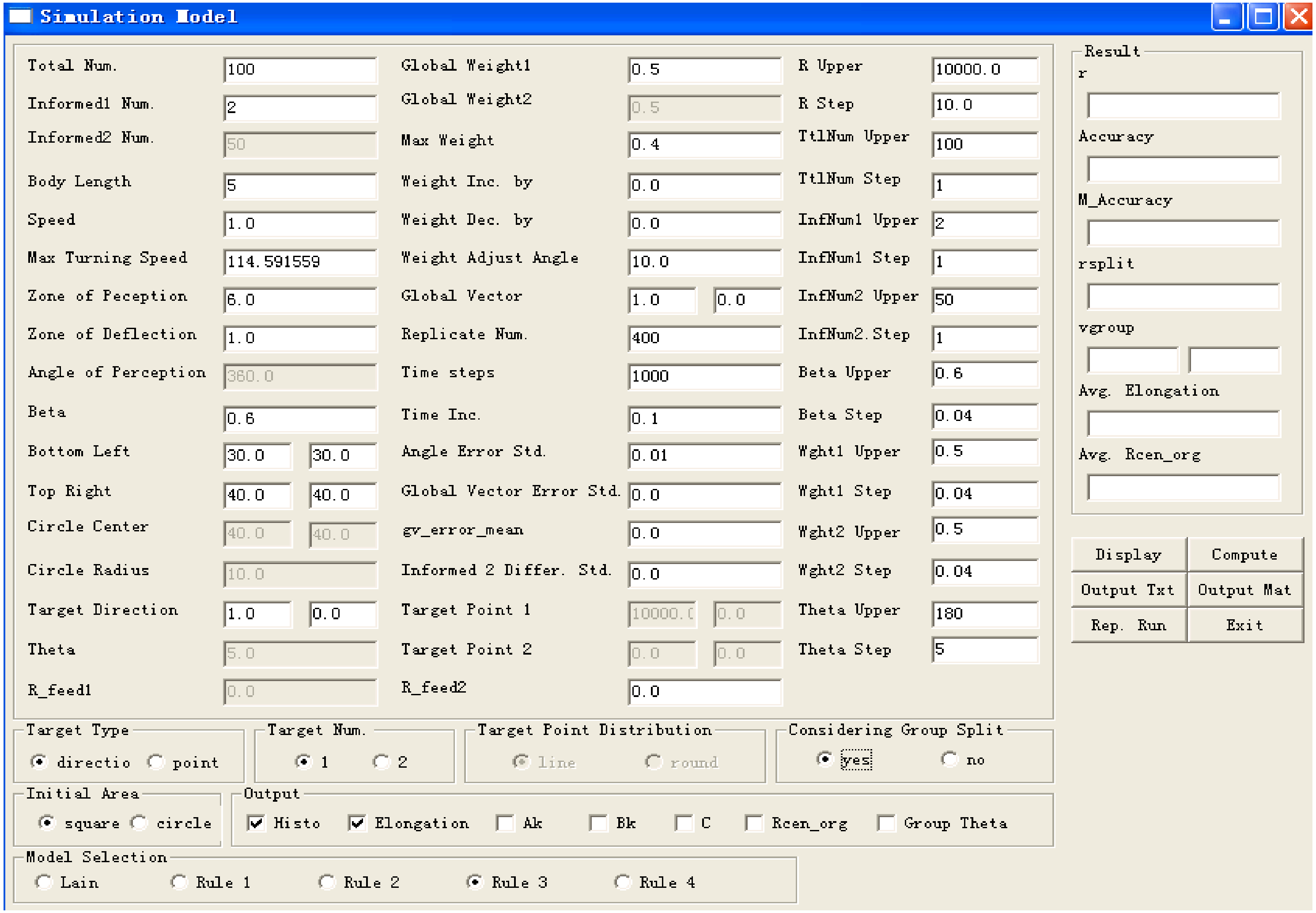


(c) 自发的同步掌声



Panic Escape **Clapping Synchronization**

Simulation Platform of Collective Behaviors



J. Lü, **J. Liu, I. D. Couzin, S. A. Levin, Emerging collective behaviors of animal groups, WCICA, 2008, pp. 1060-1065.**

Theoretical Analysis

- **Simplification of Vicsek model by linearization:**

$$\theta_i(t+1) = \frac{1}{|N_i(t)|} \sum_{j \in N_i(t)} \theta_j(t)$$

A. Jadbabie, J. Lin, and A.S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules, IEEE Trans. Automat. Contr., 2003, 48(6): 988-1001

- **Analysis of this model is based on Wolfowitz Theorem:**

Given a set of finite number of SIA matrices, if any finite products generated from this set is SIA, then any infinite products generated from this set is convergent

J. Wolfowitz, Products of indecomposable, aperiodic, stochastic matrices, Proc. Amer. Math. Soc, 1963, 15: 733-737.

Continuous-Time vs. Discrete-Time

- **Fixed Topology**
  - **Continuous-Time:**

    Lyapunov Function, Eigenvalue Computation
  - **Discrete-Time:**

    Lyapunov Function, Eigenvalue Computation

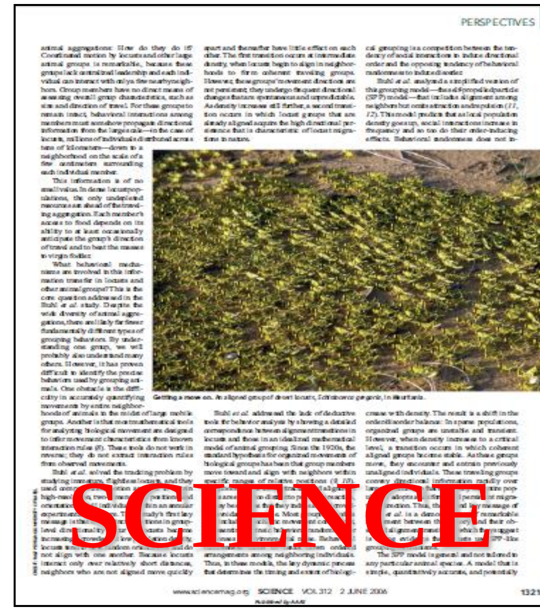
- **Switching topology**
  - **Continuous-Time:**

    Lyapunov Function
  - **Discrete-Time: [Nonexistence of quadratic Lyapunov function]**

    Convex Analysis, Stochastic Approximation, Graph Theory

A. Olshevsky, J. N. Tsitsiklis, On the nonexistence of quadratic Lyapunov function for consensus algorithms, *IEEE* Trans. Automat. Contr., 2008, 53(11): 2642-2645.

Experimental Observation



- Fish Migration / Motion
- Locust Breeding
- Bird Migration
- ……

Control of MAS: Main Approaches

- Leader-Follower Control
- Coordinated Control

  Swarming, Consensus, Flocking, …
- Data Traffic Control

  Shortest-Path, Betweenness, …
- Switch Control

  Switch Rule, Switch Times, …
- Pinning Control

  Selective Scheme, Network Structure, Node Dynamics, …
- Intervention
- …

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- Introduction
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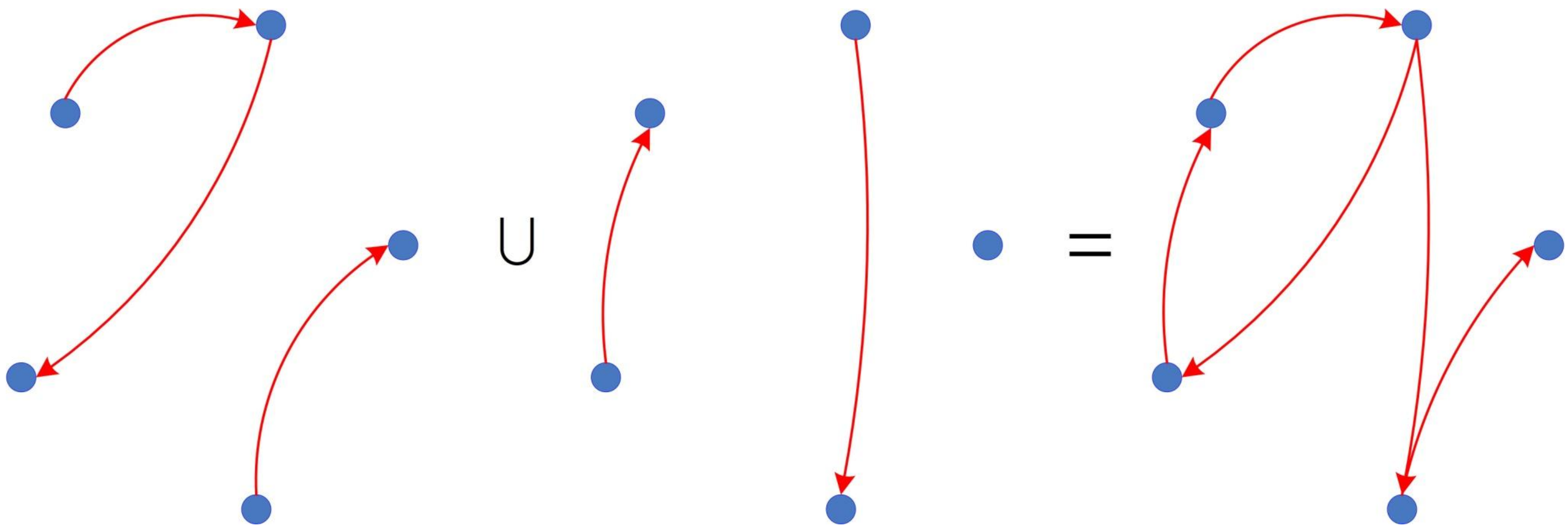
- **Consensus of MAS: Several Typical Cases**
- Conclusions

Consensus of MAS: Several Typical Cases

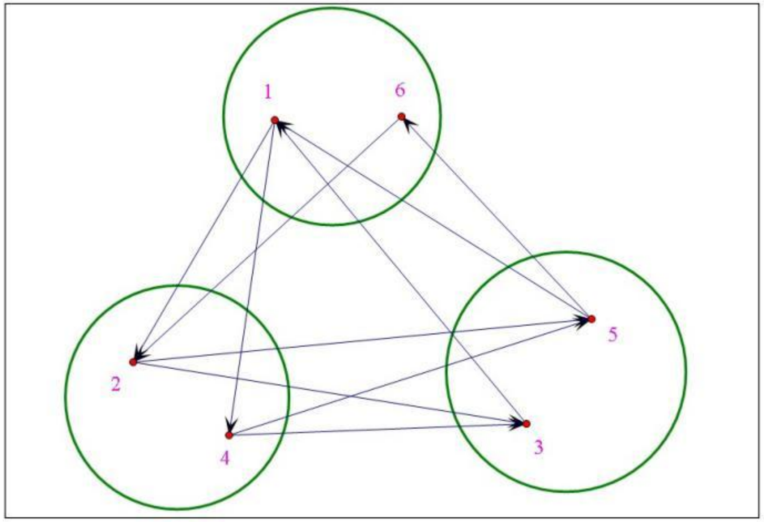
- **CASE I: Cluster Consensus of Discrete-Time MAS**
- **CASE II: Consensus of Discrete-Time MAS with Nonlinear Transmission**
- **CASE III: Infinite Products of General Stochastic Matrices**

Jointly Connected Graphs



The Period of A Graph

- **For any node of a strongly connected graph, the GCD (<u>Greatest Common Divisor</u>) of the lengths of all paths starting from this node and ending in the same node is called <u>the period of this graph</u>.**
- **The following graph has period 3.**



CASE I: Cluster Consensus of Discrete-Time MAS

- **Model Description**

$$x_i(t+1) = \sum_{j=1}^{N} a_{ij}(t) x_j(t), \quad i = 1, 2, \ldots, N. \qquad \textbf{(2)}$$

- **Definition of Cluster Consensus: If there exist** k **different sets** $\{V_r\}_{r=0}^{k-1}$ **with** $V_i \neq V_j$ **for any** $i \neq j$ **and** $\bigcup_{r=0}^{k-1} V_r = V$**, such that**

$$\lim_{t\to\infty} | x_i(t) - x_j(t) | = 0, \quad \forall i, j \in V_r.$$

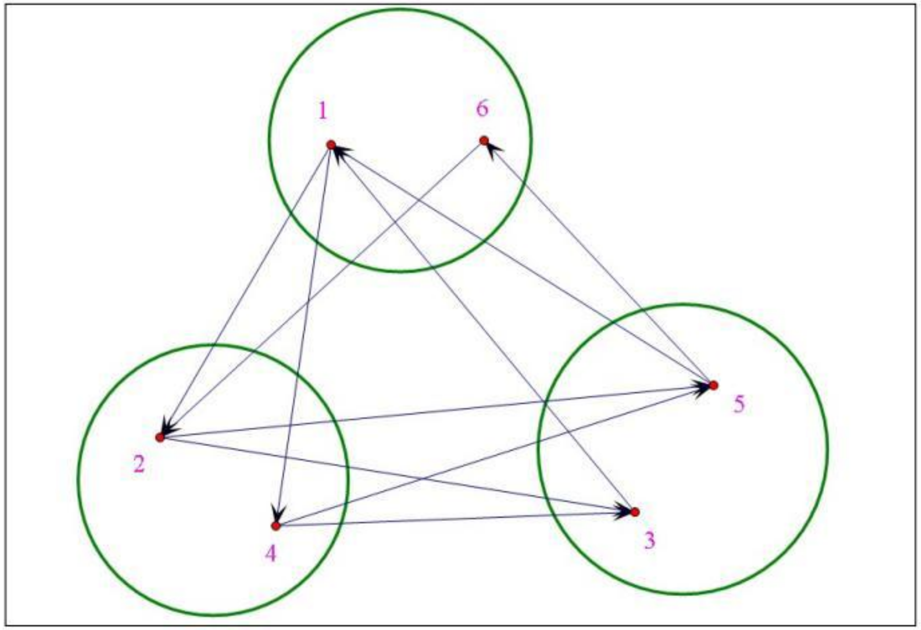
Then (2) reaches cluster consensus.

Two Basic Questions

- ◆ **How to determine the network clusters？**

- ◆ **What conditions can MAS reach consensus？**

◆ **Cluster Factorization Algorithm: Given a strongly connected graph $G=\{V,E\}$ with period d. For a given node $i \in V$ and another node $j \in V$, let the length of a path from node i to node j be d_j. If $d_j \equiv r(\bmod\ d)$, then $j \in V_r$ where $0 \le r < d$.**



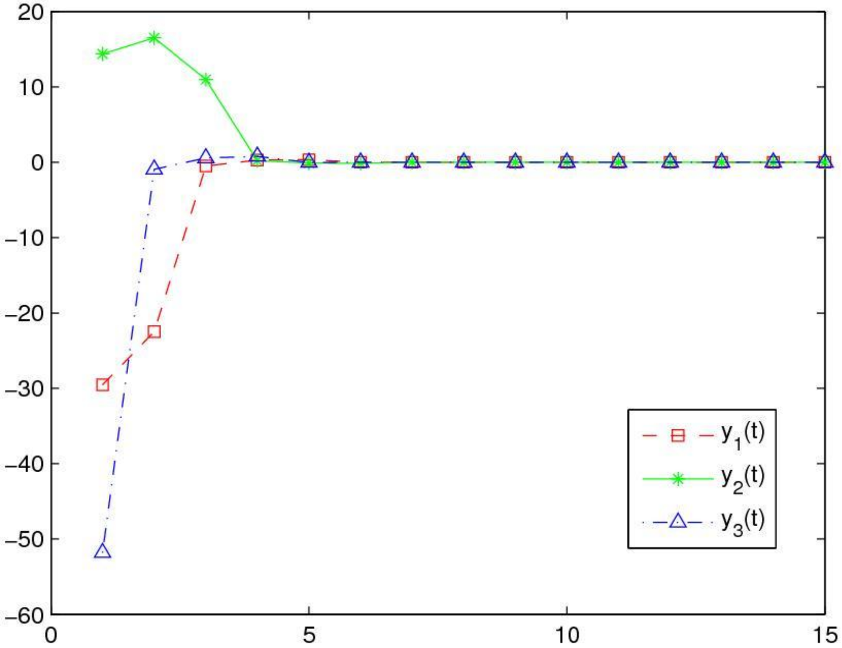
A Typical Example of Cluster Factorization

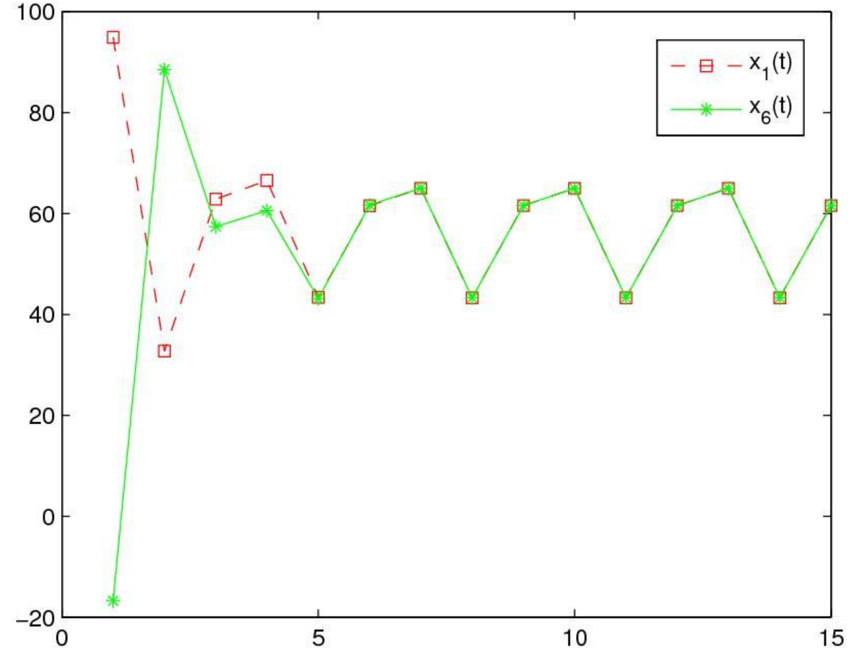
Main Result

Theorem 1: If $G(A)$ is fixed and has period d , $\inf_{a_{ij}(t)>0, t\geq 0} a_{ij}(t) \geq \alpha$, then MAS (2) reaches d-cluster consensus.

Y. Chen, J. Lü, F. Han, X. Yu, On the cluster consensus of discrete-time multi-agent systems, Syst. Contr. Lett., 2011, 60(7): 517-523.

An Example





6 agents can be classified into the following three clusters: V0 = {1, 6}, V1 = {2, 4}, V2 = {3, 5} and also reach cluster consensus .

CASE II: Consensus of Discrete-Time MAS with Nonlinear Transmission

Model Description:

$$x_i(t+1)=\sum_{j=1}^{N} a_{ij}(t) f_{ij}(x_j(t-\tau_j^i(t))), \quad i=1,2,\ldots,N. \quad \textbf{(2)}$$

$f_{ij}(\bullet)$ **Nonlinear Interaction**

$a_{ij}(t)$ **Coupling Coefficients**

$\tau_j^i(t)$ **Time Delays**

One Basic Question

◆**Definition of consensus**:

$$\lim_{t\to\infty} \| x_i(t) - x_j(t) \| = 0$$

for any $i, j \in V$ **.**

◆**Basic Question**: **What kind of nonlinear functions, time delays and topology structures can make MAS (2) reach consensus?**

A Class of Nonlinear Functions

f **belongs to** F **if the following conditions are satisfied:**

1. f **is continuous and** $f \in \boldsymbol{R}^m \to \boldsymbol{R}^m$
2. f **is defined on some convex set** $\mathcal{B} \subseteq \boldsymbol{R}^m$ **and** $f(x) \in B$ **when** $x \in B$ **.**
3. **There exists a bounded convex set** $U \subseteq B$ **such that** $f(x) = x$ **for** $x \in U$ **and** $f(x) \neq x$ **for** $d(f(x), U) < d(x, U)$ **.**

Assumptions

A1. $f_{ij} \in F$ **for any** $i, j \in V$ **, and** $\{f_{ij}\}_{i,j=1}^{n}$ **share two common sets** B **and** U **.**

A2. $\{G(t)\}_{t=0}^{\infty}$ **is jointly connected.**

A3. $0 \le \tau_j^i(t) < B$ **for any** $i \neq j$ **,** $\tau_i^i(t) = 0$ **for any** $i \in V$**.**

A4. $a_{ij}(t) \ge 0,\ a_{ii}(t) > 0,\ \sum_{j=1}^{N} a_{ij}(t) = 1$ **for any** $i, j \in V$ **,** $\inf_{a_{ij}(t)>0, t\ge 0} a_{ij}(t) \ge \alpha$ **for some** $\alpha \in (0, \frac{1}{2}]$ **.**

Main Result

Theorem 2: **If the above Assumptions 1-4 hold for MAS (2), then MAS (2) reaches consensus.**

Y. Chen, J. Lü, Z. Lin, Consensus of discrete-time multi-agent systems with nonlinear transmission, Automatica, 2012. (Provisionally Accepted)

An Example

Consider the following MAS

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t). \end{cases}$$

The controller is given by

$$u_i(t) = -\gamma_0 v_i(t_k) + \gamma_1 \sum_{j=1,\, j\neq i}^{n} a_{ij}(t_k)(f_{ij}(x_j(t_k)) - x_i(t_k))$$

where $t \in [t_k, t_{k+1})$ **.**

The above MAS can be transformed into MAS (2) via a simple linear transformation and consensus criteria can be obtained subsequently.

CASE III: Infinite Products of General Stochastic Matrices with Its Application to Consensus of MAS

- **Background**: **Second order system is one of the basic mechanical systems. And the interconnected second order MAS with dynamical topology is a fundamental class of MAS.**

- **Question**: **Second order MAS with fixed topology can be analysed by calculating eigenvalues or constructing smooth Lyapunov functions. How to analyse second order MAS with dynamical topology?**

Some Known Results (1)

◆Common Quadratic Lyapunov Function

MAS does not exist a common quadratic Lyapunov function under some specific conditions.

A. Olshevsky, J. N. Tsitsiklis, On the nonexistence of quadratic Lyapunov function for consensus algorithms, IEEE Trans. Automat. Contr., 2008, 53(11), 2642-2645.

Some Known Results (2)

For MAS, if there exist two system matrices with different left perron eigenvector, there does not exist a convex, smooth, and closed common Lyapunov function.

R. K. Brayton and C. H. Tong, Stability of dynamical systems: Aconstructive approach, IEEE Trans. Circits Syst., 1979, 26(4): 224-234.

Mathematical Model

◆Model Description

$$\begin{cases} x_i(t_{k+1}) = x_i(t_k) + h_k x_i(t_k), \\ v_i(t_{k+1}) = v_i(t_k) + h_k u_i(t_k), \end{cases} \quad \textbf{(3)}$$

where

$$u_i(t_k) = \alpha \sum_{j \in N_i(t_k)} a_{ij}(t_k)(x_j(t_k) - x_i(t_k)) + \beta \sum_{j \in N_i(t_k)} a_{ij}(t_k)(v_j(t_k) - v_i(t_k)).$$

Here, $A_k = (a_{ij}(t_k))_{i,j=1}^{N}$ **are stochastic matrices.**

An Open Problem

◆**Definition of consensus**:

$$\lim_{t\to\infty} \| x_i(t_k) - x_j(t_k) \| = 0,$$

$$\lim_{t\to\infty} \| v_i(t_k) - v_j(t_k) \| = 0,$$

for any $i, j \in V$ **.**

◆**Basic Question**: **What kind of switching topology and sampling interval can guarantee the consensus of MAS (3) ?**

Assumptions 1 and 2

◆ **Consensus** **without constraint on self-loops**

Assumption 1: For the matrices $A_k = (a_{ij}(t_k))_{i,j=1}^{N}$ **, there is** $\inf_{a_{ij}(t)>0, t\geq 0} a_{ij}(t_k) \geq \gamma$ **for some** $\gamma \in (0,1)$ **,** $a_{ij}(t_k)^2 + a_{ji}(t_k)^2 \neq 0$ **for any** $k \geq 1$ **.**

Assumption 2: The discretization step length h_k **satisfy**

$$h_k \in [\frac{1}{(2+\gamma)\beta - \mu^{-1}}, \frac{1-\delta}{\beta + \mu^{-1}}]$$

where $\mu = \frac{\beta}{\alpha}$ **and** $\delta \in (0,1)$ **.**

Main Result (1)

Theorem 3：If Assumptions 1 and 2 hold, then the MAS (3) reaches consensus.

Y. Chen, J. Lü, X. Yu, Z. Lin, Infinite products of a class of general stochastic matrices with their application to consensus of multi-agent systems, SIAM Journal on Control & Optimization, 2012. (Second Review, Minor Modification)

Assumptions 3 and 4

◆ **Consensus with <u>constraint on the self-loops</u>**

Assumption 3: Each A_k is diagonal dominant and

$$\eta(A_k) < \min\{1 - 2\frac{\alpha}{\beta^2}, 1 - \delta - \frac{\alpha}{\beta^2}(2-\delta)\}$$

for some $\delta \in (0,1)$, where

$$\eta(A) = \frac{1}{2}\max_{i,j}(a_{ii} + a_{jj} - a_{ij} - a_{ji} + \sum_{k \neq i,j} |a_{ik} - a_{jk}|)$$

Assumption 4: The discretization step length h_k satisfying

$$h_k \in [\frac{\delta}{(1-\eta(A_k))\beta - 2\mu^{-1}}, \frac{2-\delta}{\beta(1+\eta(A_k))}]$$

where $\mu = \frac{\beta}{\alpha}$ and $\delta \in (0,1)$.

Main Result (2)

Theorem：If Assumptions 3 and 4 hold, then the MAS (3) reaches consensus.

Y. Chen, J. Lü, X. Yu, Z. Lin, Infinite products of a class of general stochastic matrices with their application to consensus of multi-agent systems, SIAM Journal on Control & Optimization, 2012. (Second Review, Minor Modification)

Main Ideas of Proof

- It is often difficult to construct a traditional smooth Lyapunov function to analyse the stability of MAS with switching topology

- We construct a polytope in 2N dimensional space and demonstrate that the network dynamics contracts along the polytope

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Conclusions

- Some new results on consensus of discrete-time MAS are introduced
- Some new methods for coping with dynamical topology and nonlinear interactions are proposed
- These methods can be further generalized to consensus of other MAS

Some Future Works

- Further investigation of non-convex MAS models, trying to find a more universal method to tackle this kind of problems
- How to cope with dynamical topology effectively?
- How to cope with complex nonlinear interactions efficiently?
- How to cope with environment uncertainty?

Thank You Very Much !

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